Lightning Talks III
Tech Topology Conference

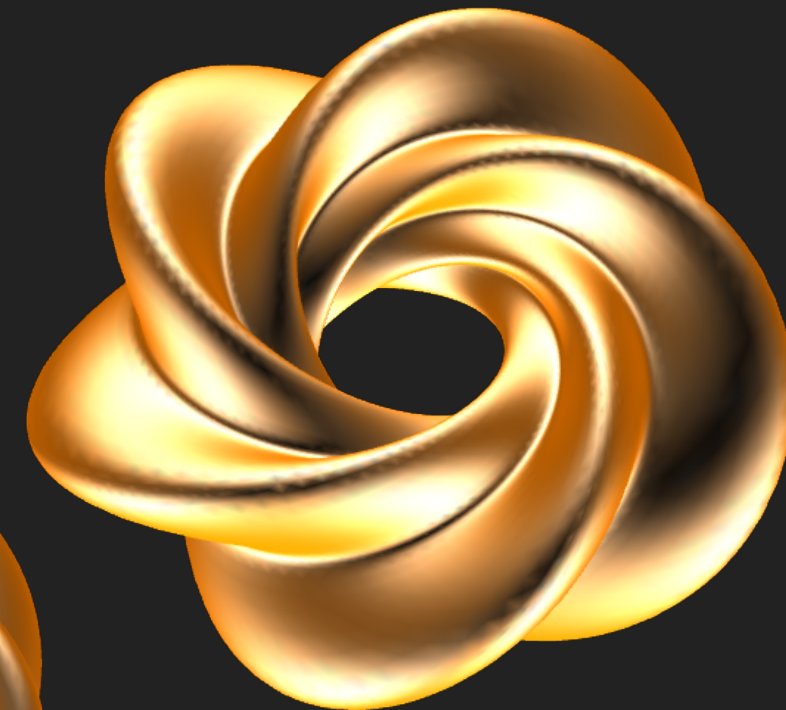
December 9, 2018

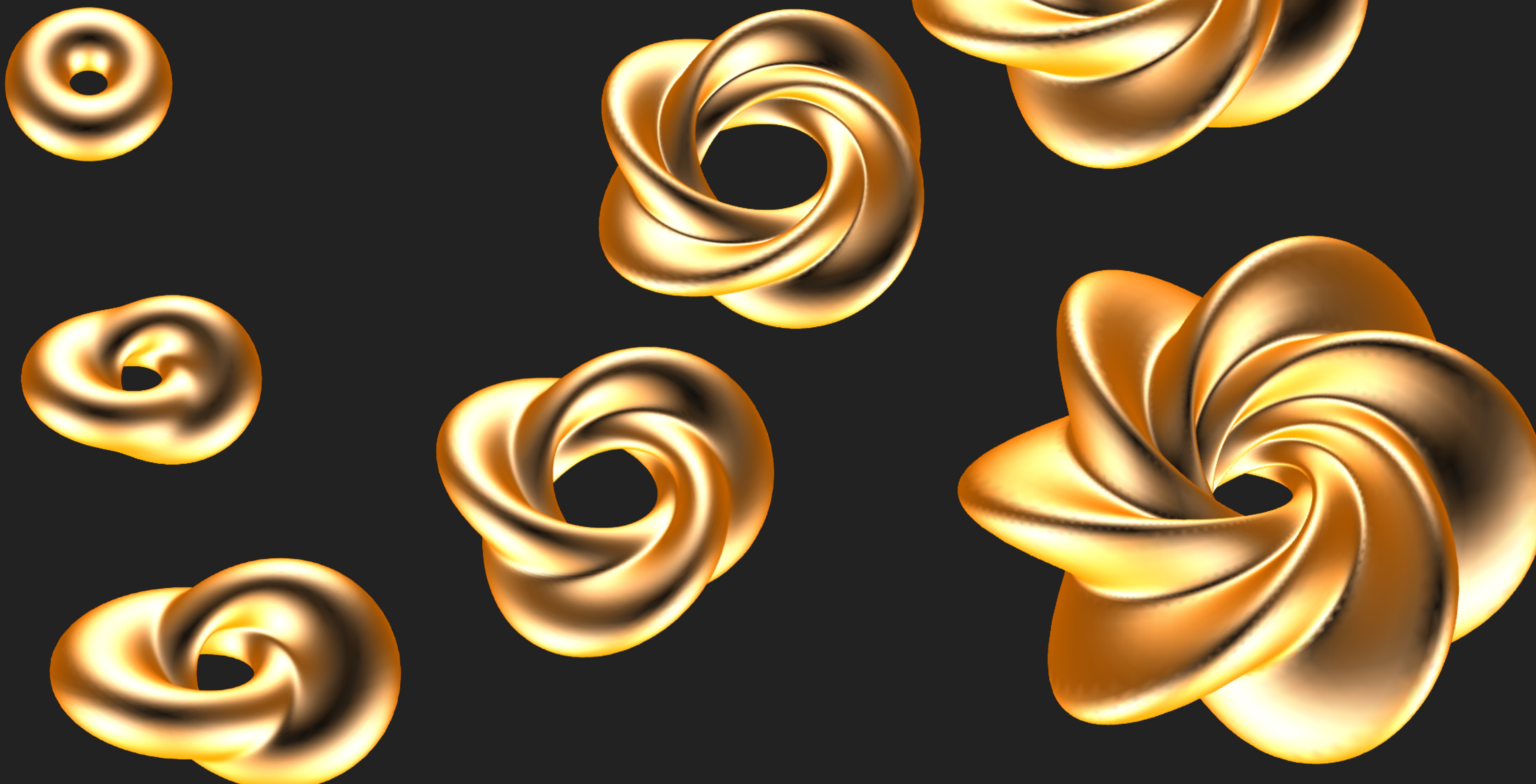
STEVE TRETTEL
UC Santa Barbara

PROJECTIVE GEOMETRY,
COMPLEX HYPERBOLIC SPACE &

GEOMETRIC TRANSITIONS

DEFORMATIONS ARE INTERESTING





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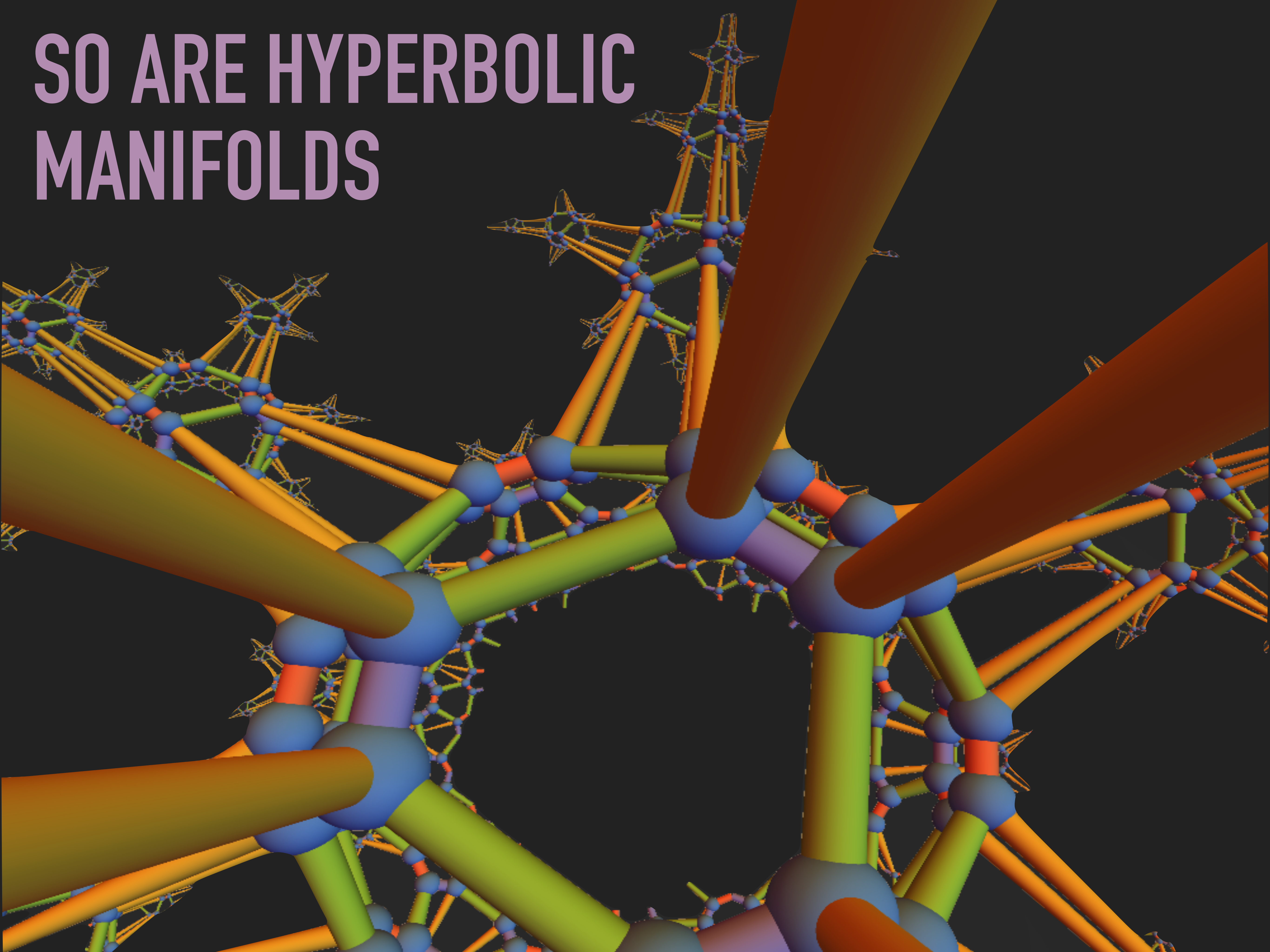
SO ARE HYPERBOLIC MANIFOLDS



Q: Is there a Deformation theory of hyperbolic n-Manifolds ?

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A: NO ☹

MOSTOW RIGIDITY

A: YES

THERE'S TWO!

$$\mathrm{Aut}(\mathbb{RP}^n)$$

$$\cup$$

$$\mathrm{Isom}(\mathbb{H}^n)$$

$$\cap$$

$$\mathrm{Isom}(\mathbb{H}^n_{\mathbb{C}})$$

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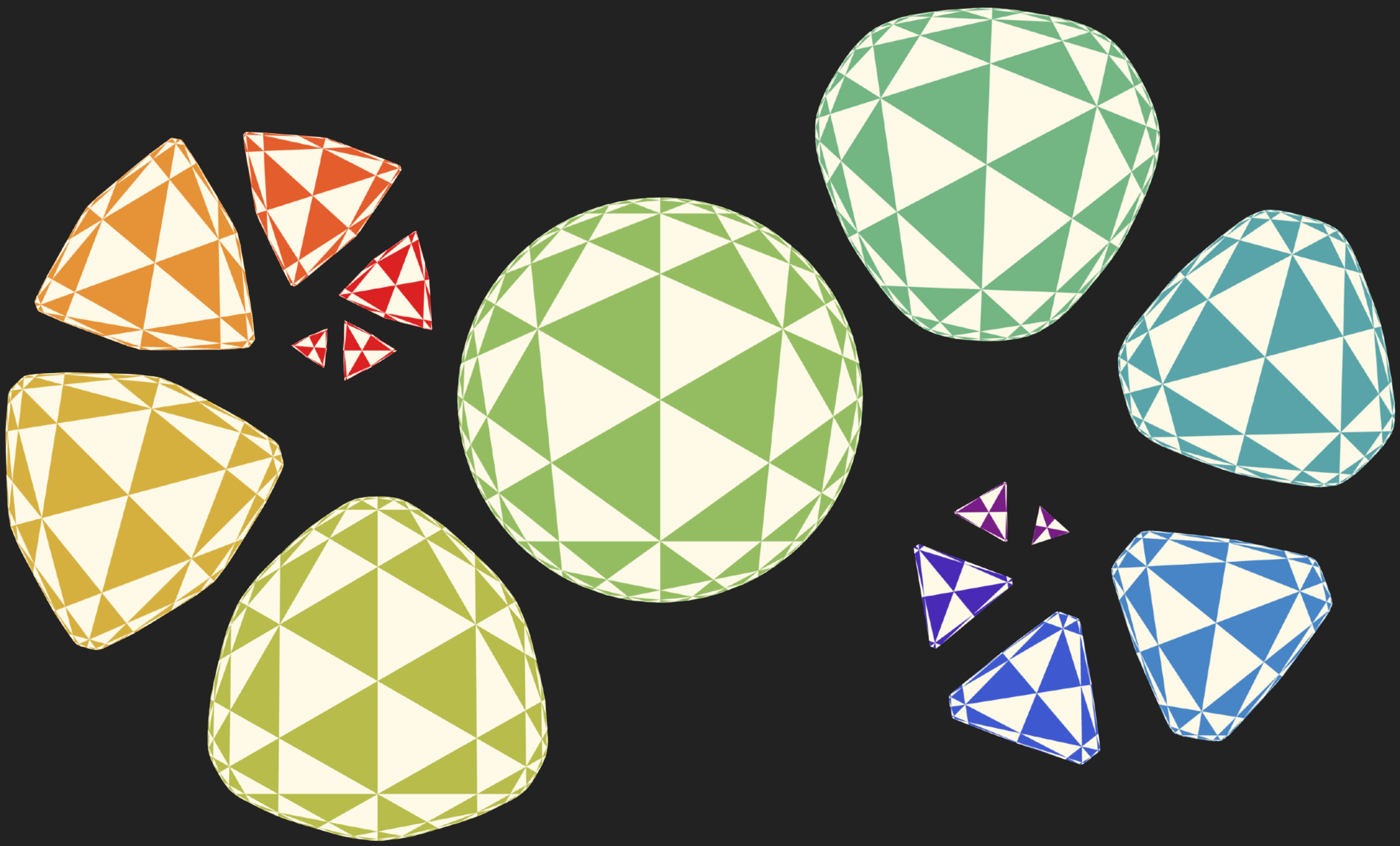

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ARE THESE RELATED?

YES

The representation varieties have the **same dimension**.

NO

The geometries have **different dimensions**.

$\mathbb{H}^n_{\mathbb{C}}$? $\mathbb{RP}^n$

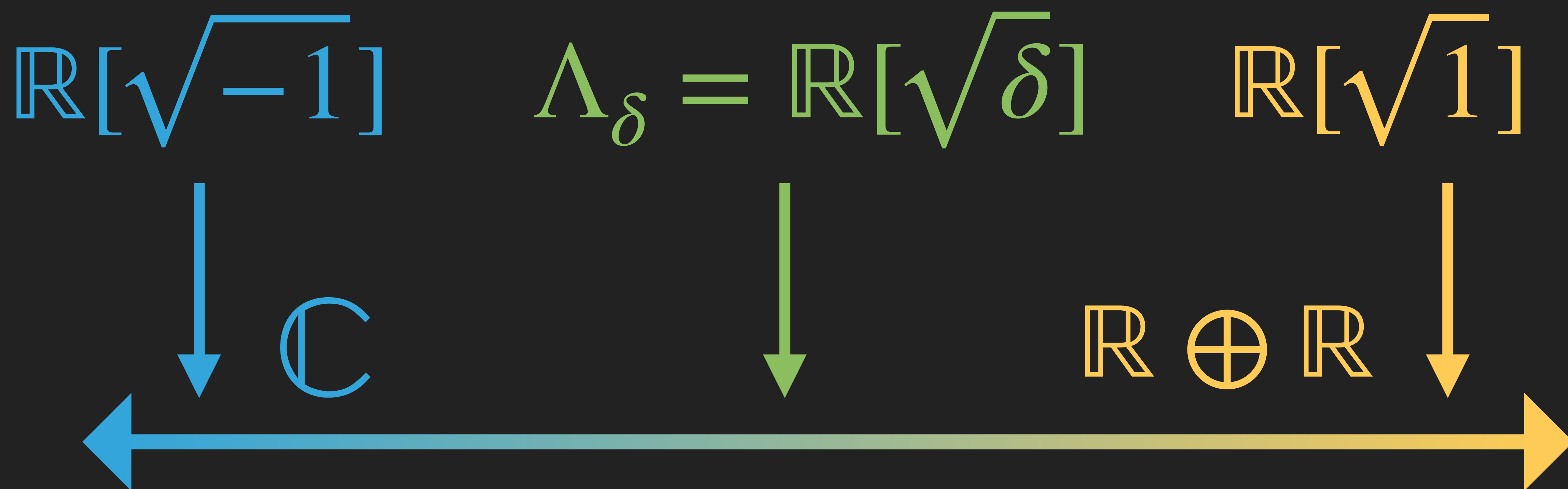
YES!

THEOREM (T-18)

Real Projective Space is a 'shadow' of a higher dimensional geometry.

This geometry is a deformation of Complex Hyperbolic Space.

DEFORM $\mathbb{C}$



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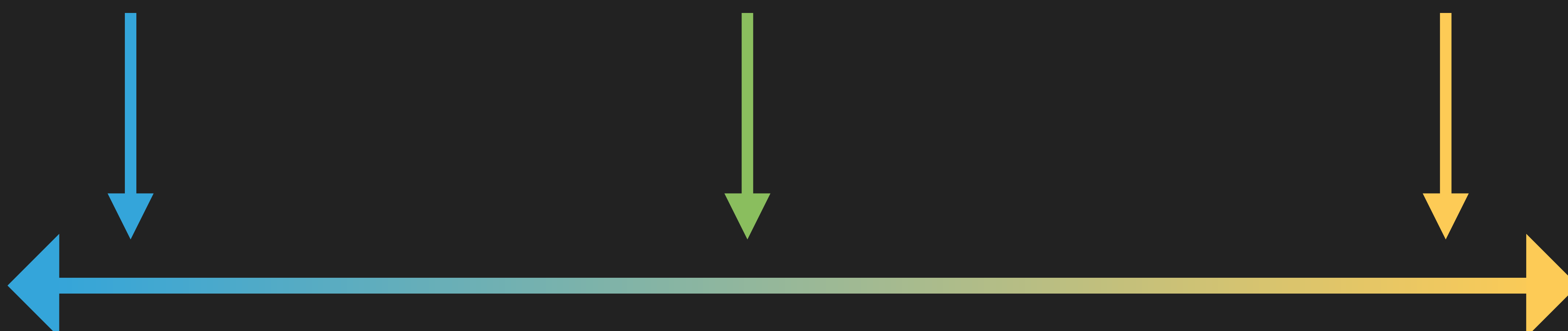
DEFORM $\mathrm{Isom}(\mathbb{H}^n_{\mathbb{C}})$

$\mathrm{SU}(n,1;\mathbb{C})$ $\mathrm{SU}(n,1;\Lambda_\delta)$ $\mathrm{SU}(n,1;\mathbb{R}\oplus\mathbb{R})$



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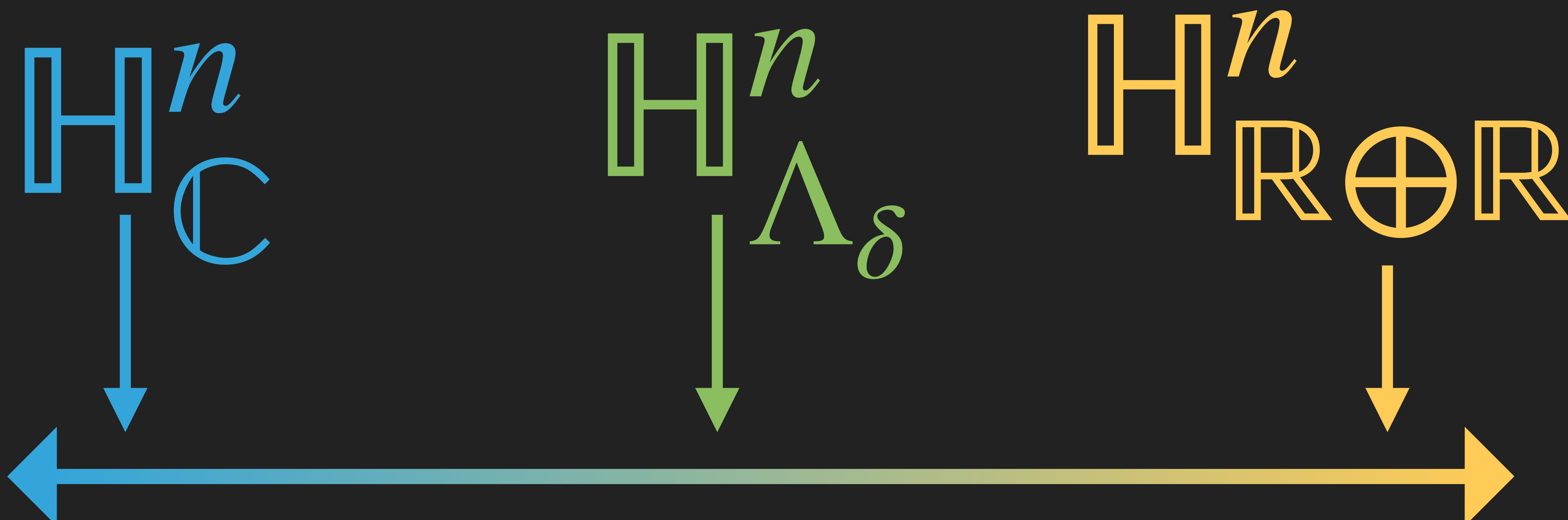
DEFORM $\mathbb{H}^n_{\mathbb{C}}$



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UNDERSTAND $\mathbb{H}^n_{\mathbb{R}\oplus\mathbb{R}}$

THEOREM (T-18)

$\mathbb{H}^n_{\mathbb{R}\oplus\mathbb{R}}$ embeds nicely in $\mathbb{R}\mathsf{P}^n \times \mathbb{R}\mathsf{P}^n$
and has automorphisms $\cong \mathsf{SL}(n+1;\mathbb{R})$

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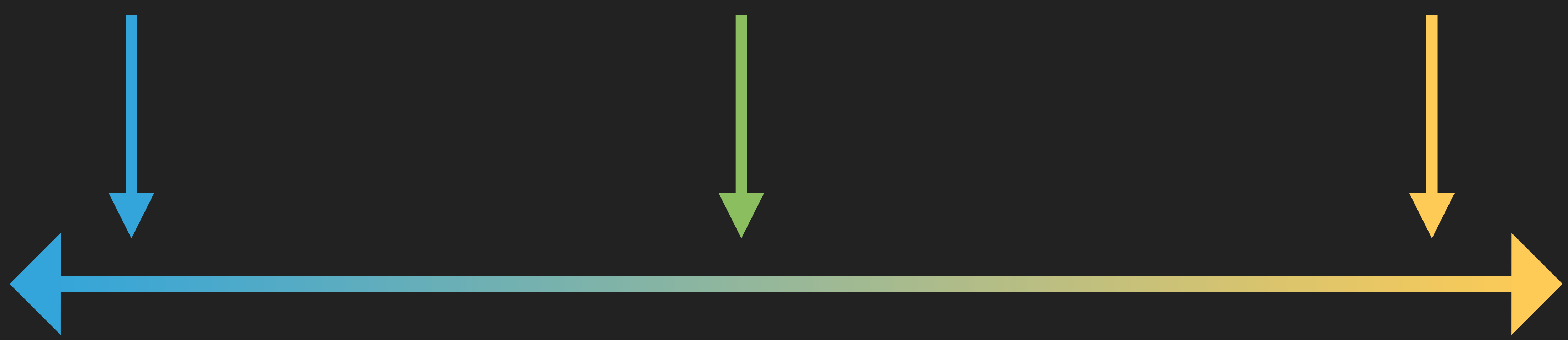
$\mathbb{H}^n_{\mathbb{C}}$ Holonomy

1 parameter family of deformations

$\mathbb{R}\mathrm{P}^n$ Holonomy



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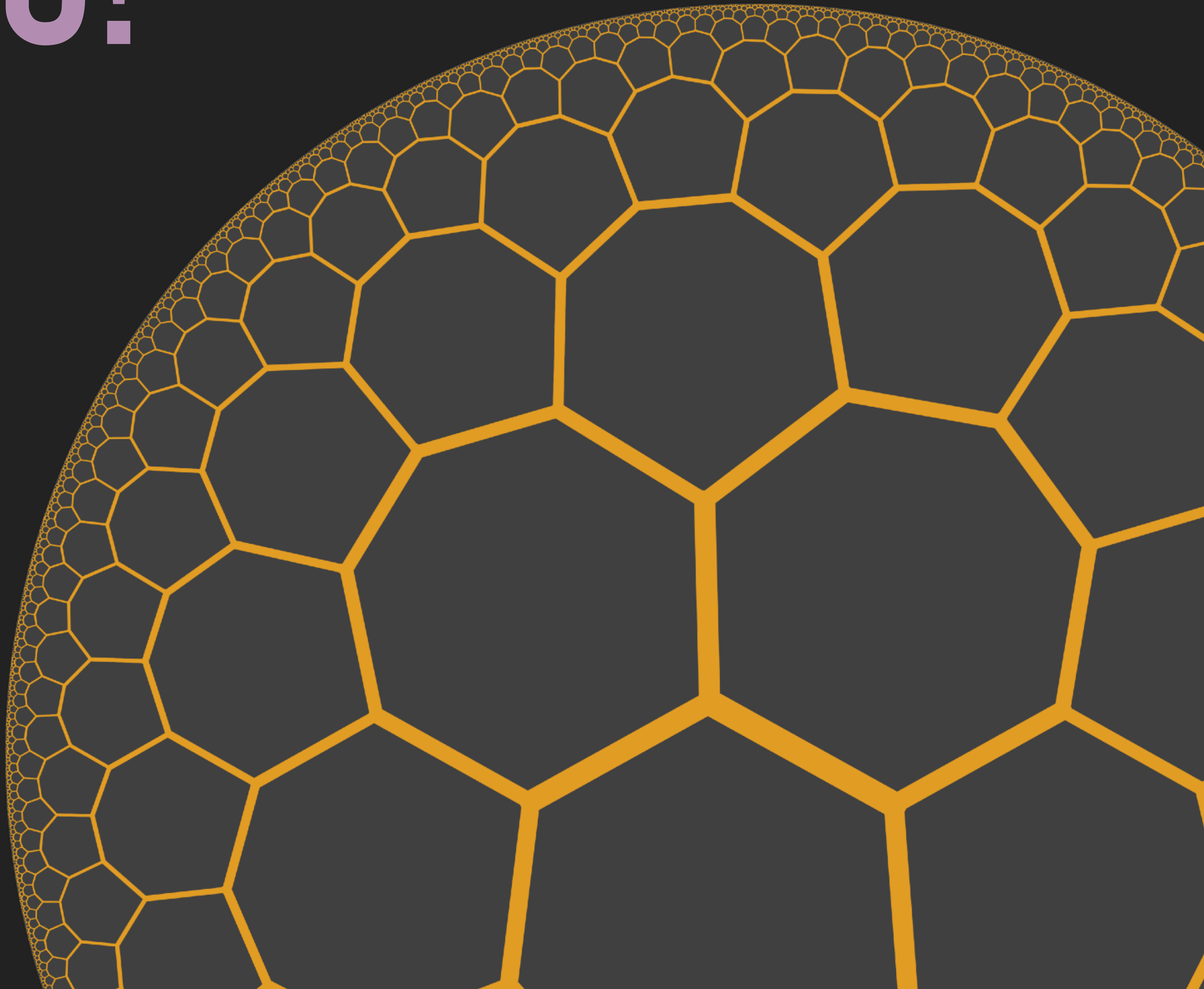
THANKS!



Symmetry and Localization

Melissa Zhang

Boston College

Tech Topology 2018

What is localization?

What is localization?

Scenario:

- Topological space X

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- $\mathbb{Z}_p$ action on X (p prime)

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Theorem (Classical localization theorem)

$$H^*(X;\mathbb{Z}_p) \rightrightarrows H^*(X^{\mathrm{fix}};\mathbb{Z}_p).$$

What is localization?

Scenario:

- Topological space X
- $\mathbb{Z}_p$ action on X (p prime)
- Fixed point set X^{fix}

Theorem (Classical localization theorem)

$$H^*(X;\mathbb{Z}_p) \rightrightarrows H^*(X^{\text{fix}};\mathbb{Z}_p).$$

Corollary (Classical Smith inequality)

Under certain conditions,

$$\dim H^*(X;\mathbb{Z}_p) \geq \dim H^*(X^{\text{fix}};\mathbb{Z}_p).$$

For the following, $G = \mathbb{Z}_2$.

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Theorem (Seidel-Smith 2010)

- *Under certain conditions,* $LFH(M, L_0, L_1) \rightrightarrows LFH(M^{\text{fix}}, L_0^{\text{fix}}, L_1^{\text{fix}})$.
- *Application:* $Kh_{symp}(\tilde{L}) \rightrightarrows Kh_{symp}(L)$.

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Conjecture (Seidel-Smith)

- $Kh \cong Kh_{symp}$?
- $\dim Kh(\tilde{L}) \geq \dim Kh(L)$?

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Question (Lidman):

Is it possible to recover

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Answer (Lipshitz-Treumann)

Partial "yes." Under certain conditions,

$$HH_*(M \otimes^L_A M) \rightrightarrows HH_*(M).$$

Use bordered Floer homology.

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Theorem (Cornish 2016)

If $\sigma \in B_n$, then in annular grading $k = n - 1$,

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- *Combinatorial proof*
- *Khovanov analogue of Hendricks's $\widehat{HFK}(S^3, \tilde{K} \cup \tilde{U})$ vs. $\widehat{HFK}(S^3, K \cup U)$ result*

Recall:

Conjecture (Seidel-Smith)

$$\dim Kh(\tilde{L}) \geq \dim Kh(L).$$

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... And what about $G = \mathbb{Z}_p$?

Now $G = \mathbb{Z}_p$. (!!!)

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- Uses Lawson-Lipshitz-Sarkar's Burnside functor construction of the Lipshitz-Sarkar Khovanov stable homotopy type.
- Also holds for odd versions of all theories involved.
- Framework also generalizes $AKh(\tilde{L}) \rightrightarrows AKh(L)$ to prime-periodicities.

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Theorem (Lidman-Manolescu 2016)

If $\tilde{Y}$ is a rational homology sphere and $\tilde{Y} \to Y$ is a p-sheeted regular cover,

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Also see work of

- Politarczyk, Borodzik-Politarczyk
- Borodzik-Politarczyk-Silvero
- Boyle
- Musyt.

On the naturality of grid homology

Haofei Fan

Department of Mathematics
University of California, Los Angeles

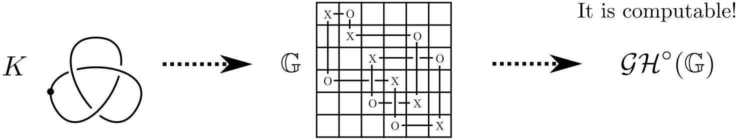
December 2018

Joint work with M. Marengon (UCLA) and M. Wong (LSU)

Grid homology

Grid homology

A link in three-sphere can be represented by a grid diagram $\mathbb{G}$. When defined for grid diagrams, the link Floer homology HFL° is usually called *grid homology* $(\mathcal{GH}^\circ)$.



Main Theorem

Question

Does a link isotopy induce a well-defined map on grid homology?

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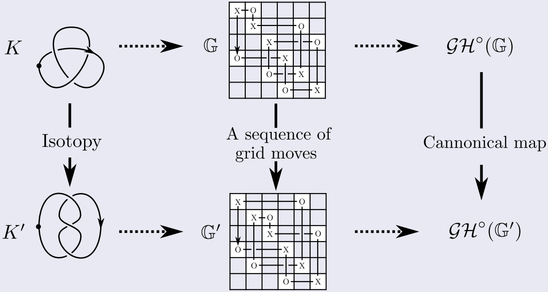
Question

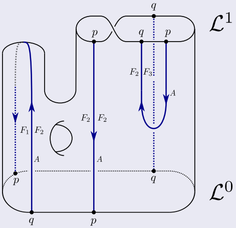
Does a link isotopy induce a well-defined map on grid homology?

Answer. Yes.

Theorem (H. Fan, M.Marengon and M. Wong (Work in progress))

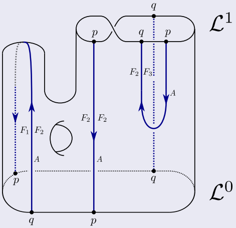


(1) Link cobordism maps are computable via grid homology

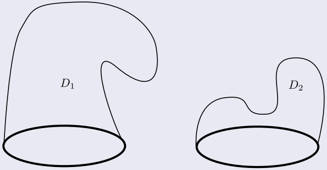


Applications Part I

(1) Link cobordism maps are computable via grid homology

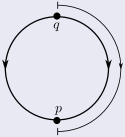


(2) Distinguish slice disks



Applications Part II

(3) Involutive knot/link/Heegaard Floer homology are computable



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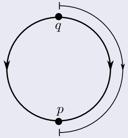


(4) Transverse/Legredrian invariants

A canonical computable isomorphism on grid homology $\phi : GH^-(\mathbb{G}) \to GH^-(\mathbb{G}')$, such that:

- $\phi(\lambda^{\pm}(\mathbb{G})) = \lambda^{\pm}(\mathbb{G}')$;
- $\phi(\theta(\mathcal{T})) = \theta(\mathcal{T}')$.

Trisecting Ozsváth Szabó Four-Manifold Invariants

William E. Olsen

December 9 2018

University of Georgia

Overview of trisections

- Suppose X is a connected, compact, oriented, smooth 4-manifold with connected boundary $Y = \partial X$.
- A trisection of X is a decomposition of X into three simple pieces. A trisection of X induces an open book on its boundary.

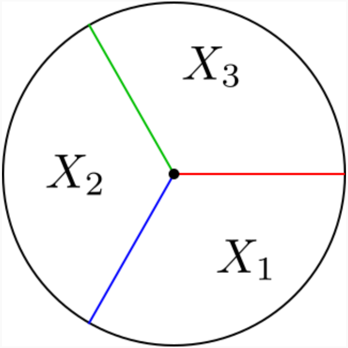
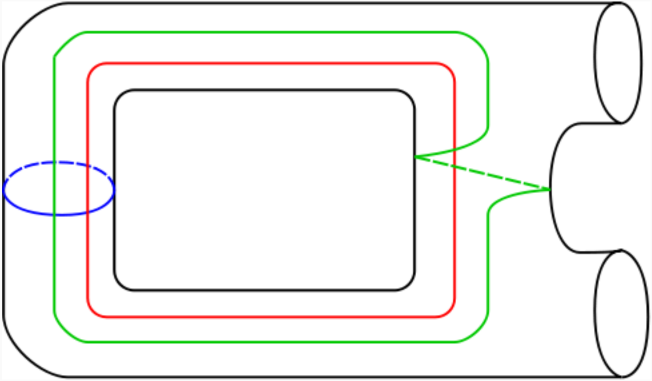


Figure 1: $X \cong X_1 \cup X_2 \cup X_3$

Diagrams for trisections

A trisected 4-manifold $X \cong X_1 \cup X_2 \cup X_3$ can be represented by a (relative) trisection diagram $\mathfrak{D} = (\Sigma, \alpha, \beta, \gamma)$ [GK16, CGPC18].



Applications to Heegaard Floer Homology?

Question

Can we use (relative) trisection diagrams to compute the cobordism maps of Ozsváth and Szabó?

$$F_{X\setminus B^4,\mathfrak{t}} : HF(S^3) \to HF(Y, \mathfrak{t}_Y)$$

Arced diagram

We decorate $\mathfrak{D}$ with arcs $\rightsquigarrow$ arrive at a new diagram $\mathfrak{D}_{arc} = (\Sigma, \alpha, \beta, \gamma, \mathfrak{a}, \mathfrak{b}, \mathfrak{c})$.

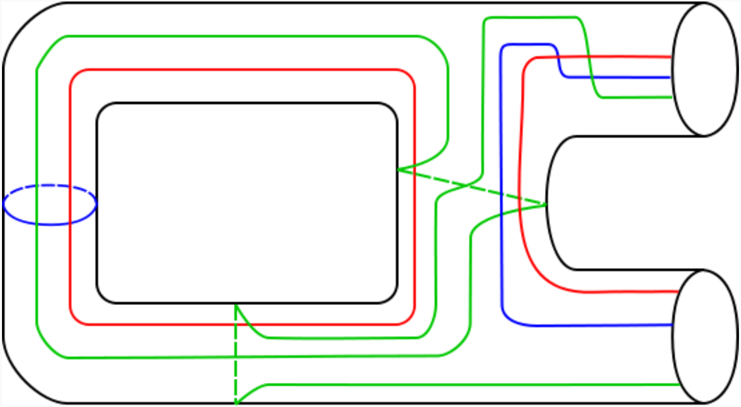
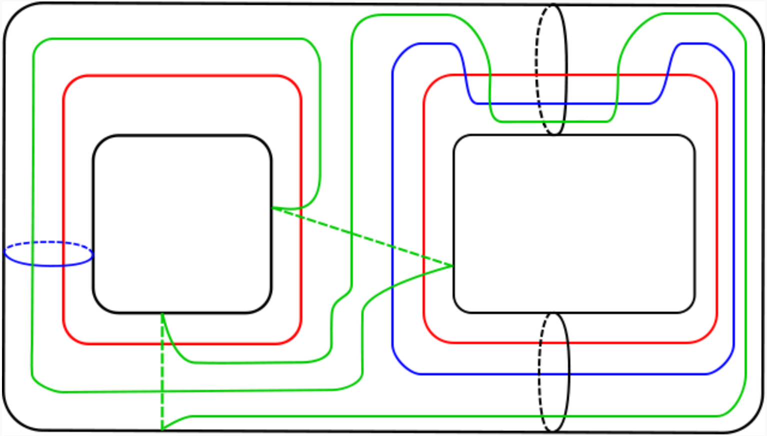


Figure 2: An arced diagram $\mathfrak{D}_{arc}$ obtained from $\mathfrak{D}$.

Gluing on the page of an open book

We glue onto $\mathfrak{D}_{arc}$ a page of the open book, and arrive at our final diagram $\underline{\mathfrak{D}} = (\underline{\Sigma}, \underline{\alpha}, \underline{\beta}, \underline{\gamma}, w)$. This diagram describes a new four-manifold $\underline{X}$.



Topology of $\underline{X}$

Proposition (Thanks to D. Gay and J. Pinzón-Caicedo)
The manifold $\underline{X}$ is diffeomorphic to

$$\underline{X} \cong \mathring{X} - \text{ 1-}\textit{handle},$$

where $\mathring{X} = X \setminus \text{interior}(X_3)$, and the 1-handle that's removed has one foot on $\#^{k_3} S^1 \times S^2 \subset \partial\mathring{X}$ and the other foot on $Y \subset \partial\mathring{X}$.

Theorem

Theorem

Suppose that $\underline{\mathfrak{D}}$ is constructed as above, and let $\mathfrak{t} \in \mathrm{Spin}^c(X)$. Then $\mathfrak{t}$ determines $\underline{\mathfrak{t}} \in \mathrm{Spin}^c(\underline{X})$ for which the following diagram commutes

$$\begin{array}{ccc}
HF(\underline{\Sigma},\underline{\alpha},\underline{\beta},w,\mathfrak{s}_{\underline{\alpha},\underline{\beta}}) & \xrightarrow{\Delta_{\underline{\alpha},\underline{\beta},\underline{\delta},\underline{\mathfrak{t}}}} & HF(\underline{\Sigma},\underline{\alpha},\underline{\gamma},w,\mathfrak{s}_{\underline{\alpha},\underline{\delta}}) \\
\downarrow \Psi_1 & & \downarrow \Psi_2 \\
HF(\#^{\ell} S^1 \times S^2, w, \mathfrak{s}) & \xrightarrow{F_{\underline{X},\underline{\mathfrak{t}}}} & HF(Y\#(\#^{k_3} S^1 \times S^2), w, \mathfrak{s}_Y \# \mathfrak{s}) \\
\uparrow & & \downarrow p \\
HF(S^3) & \xrightarrow{F_{X,\mathfrak{t}}} & HF(Y, w, \mathfrak{s}_Y)
\end{array}$$

where $F_{X,\mathfrak{t}}$ is the cobordism map defined by Ozsváth and Szabó.

References

Nickolas Castro, David Gay, and Juanita Pinzón-Caicedo.
Diagrams for relative trisections.
Pacific Journal of Mathematics, 294(2):275–305, 2018.

David Gay and Robion Kirby.
Trisecting 4–manifolds.
Geometry & Topology, 20(6):3097–3132, 2016.

Naturality of the Contact Invariant in Monopole Floer Homology under Strong Symplectic Cobordisms

Mariano Echeverria

Monopole Floer Homology

- $(Y, \mathfrak{s})$: closed oriented 3-manifold Y + spin-c structure $\mathfrak{s}$ on Y

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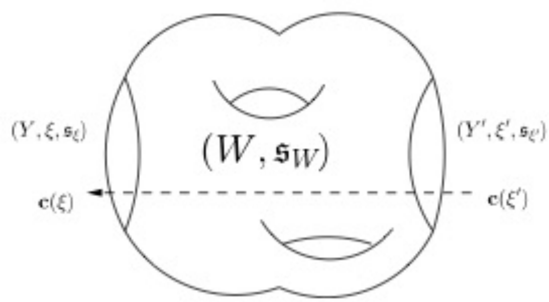
Monopole Floer Homology

- $(Y, \mathfrak{s})$: closed oriented 3-manifold Y + spin-c structure $\mathfrak{s}$ on Y
- Monopole Floer Homology produces a family of abelian graded groups $HM^{\bullet}(Y, \mathfrak{s})$
- When $(Y, \xi, \mathfrak{s}_\xi)$ is a contact manifold KMOS defined the **contact invariant**

$$\mathbf{c}(\xi) \in \widehat{HM}^{\bullet}(Y, \mathfrak{s}_\xi)$$

Naturality Problem

$$(W, \mathfrak{s}_W) : (Y, \xi, \mathfrak{s}_\xi) \to (Y', \xi', \mathfrak{s}_{\xi'})$$
$$HM^\bullet(W, \mathfrak{s}_W) : HM^\bullet(Y', \mathfrak{s}_{\xi'}) \to HM^\bullet(Y, \mathfrak{s}_\xi)$$



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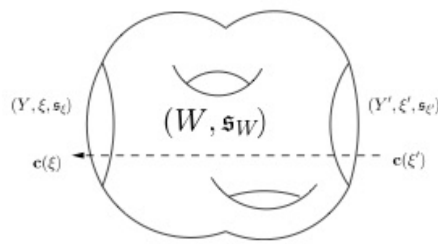
$$HM^\bullet(W, \mathfrak{s}_W) : HM^\bullet(Y', \mathfrak{s}_{\xi'}) \to HM^\bullet(Y, \mathfrak{s}_\xi)$$



Naturality Problem: For which $(W, \mathfrak{s}_W)$ is it true that
$$\widehat{HM}^\bullet(W, \mathfrak{s}_W)\mathbf{c}(\xi') = \mathbf{c}(\xi)$$

Naturality for Strong Cobordisms

Theorem (E. 2018)
Let $(W, \omega) : (Y, \xi) \to (Y', \xi')$ be a strong symplectic cobordism between two contact manifolds (Y, ξ) and (Y', ξ'). Then

$$\widehat{HM}^{\bullet}(W, \mathfrak{s}_W)\mathbf{c}(\xi') = \mathbf{c}(\xi)$$

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- Mrowka and Rollin also proved this result in an earlier, unpublished paper.
- The naturality result is with $\mathbb{Z}/2\mathbb{Z}$ coefficients
- The result is not known for Heegaard Floer in such generality.

Some Applications

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- **Corollary:** Suppose (Y, ξ) is a planar contact manifold:

1. (Ozsvath, Stipsciz and Szabo) (E.) The reduced part of the contact invariant vanishes, i.e, $[\mathbf{c}(\xi)]_{red} = 0$.
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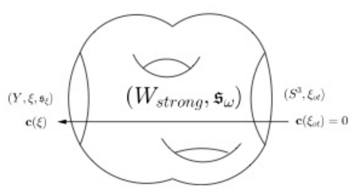
1. Find (S^3, ξ_{ot}) such that $\mathbf{c}(\xi_{ot}) = 0$.
2. If (Y, ξ) is overtwisted by Etnyre-Honda we can find $(W_{Stein}, \mathfrak{s}_\omega) : (Y, \xi, \mathfrak{s}_\xi) \to (S^3, \xi_{ot})$



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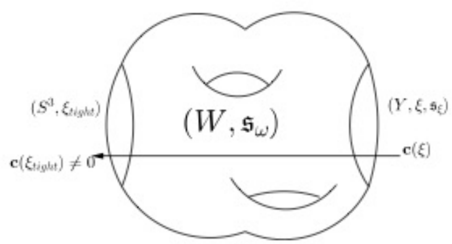
$(W = X \backslash B, \mathfrak{s}_\omega) : (S^3, \xi_{tight}) \to (Y, \xi)$



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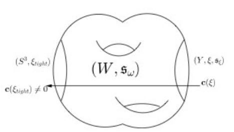

Ghiggini gave examples of weak fillings where the contact invariant vanishes, so **the naturality result cannot be naively extended**.

Thank you!

[image taken from Patrick Massot's website]

Acylindrical actions on quasi-trees

Sahana H Balasubramanya

TechToplogy Conference
December 2018

Question

Which groups admit acylindrical, non-elementary, cobounded actions on quasi-trees ?

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By a quasi-tree, I mean a connected graph quasi-isometric to a tree.

	Hyperbolic spaces	**Unbounded quasi-trees**	**Unbounded locally finite quasi-trees**
Geometric action (Proper, Cobounded)	Hyperbolic groups	Virtually free groups	Virtually free groups
Acylindrical, non-elementary cobounded action	Acylindrically hyperbolic groups	**?**	Virtually free groups

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Acylindrical, non-elementary cobounded action	Acylindrically hyperbolic groups	**Acylindrically hyperbolic groups**	Virtually free groups

Constructing Surfaces From Integer Matrices

Joshua Pankau

12/09/2018

Visiting Assistant Professor
The University of Iowa

I. Main Question

Question:

Given a positive integer matrix Q, does there exist a closed orientable surface containing a pair of filling multicurves whose intersection matrix is Q?

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Answer:

Yes.

II. Application

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Theorem[P. 2017]
If λ is an algebraic unit satisfying all the known restrictions from Thurston's construction then some power of λ is a stretch factor.

Constructing Surfaces

Example 1: Consider $\begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & 2 \\ 2 & 2 & 2 \end{bmatrix}$

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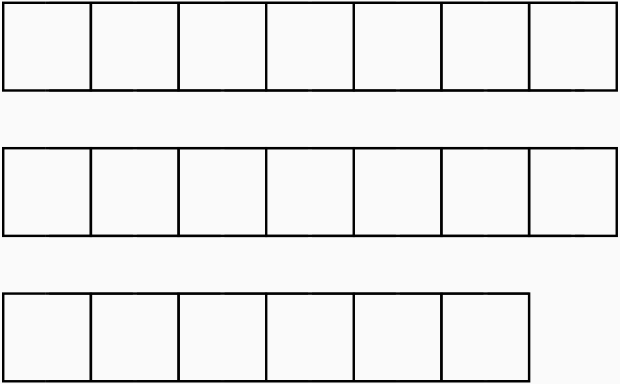

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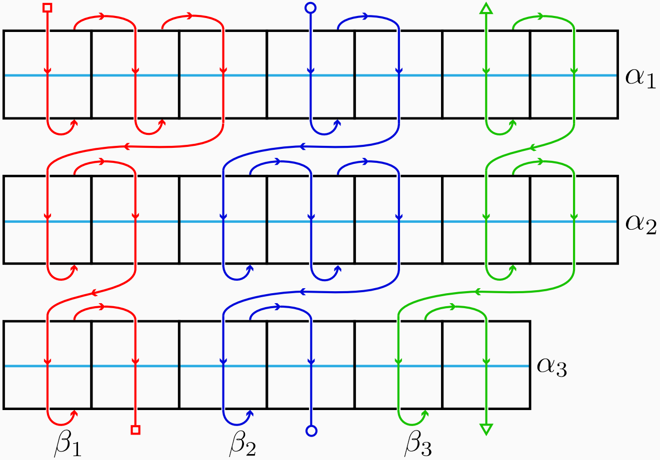

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Answer:
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Bound on Genus

Theorem:
Given an $n \times n$ matrix Q whose entries are all larger than 2. Using the configuration as in example 1, the genus of the constructed surface is $g = n^2 - n + 1$.

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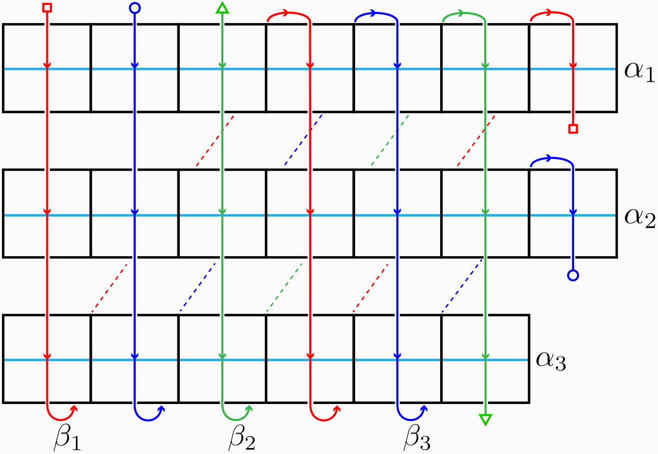
Observation:
Genus is not optimal.

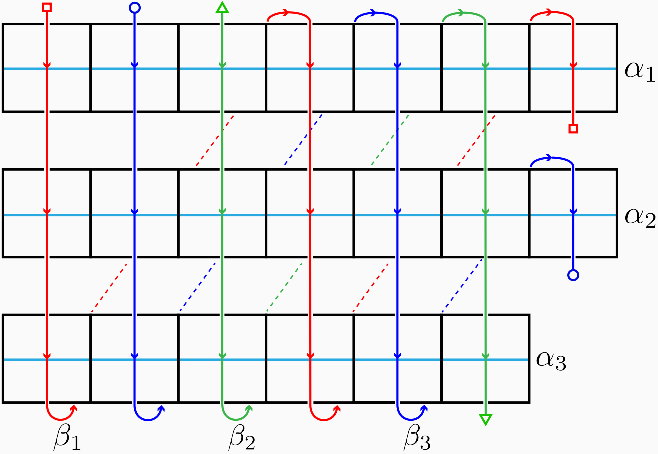
Example 2

Consider $\begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & 2 \\ 2 & 2 & 2 \end{bmatrix}$

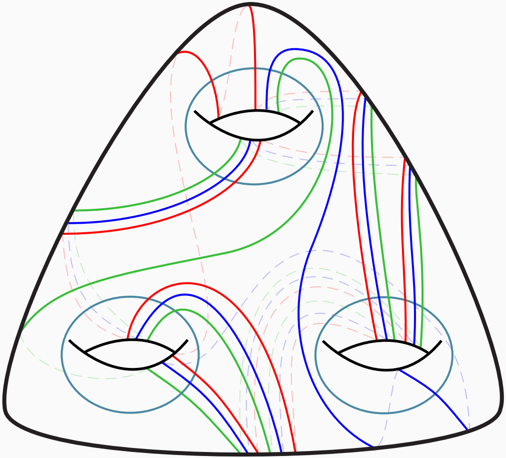


Example 2

Consider $\begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & 2 \\ 2 & 2 & 2 \end{bmatrix}$ Genus 3 surface.



Genus 3 Surface with Multicurves



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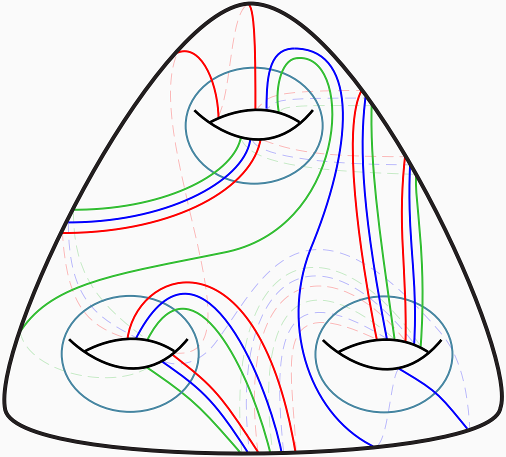

Current research question: What is the minimal genus surface obtainable?

End

Thank you!

Shadows from the pure mapping class group to the curve graph

Yvon Verberne with Kasra Rafi

University of Toronto

Pure mapping class groups

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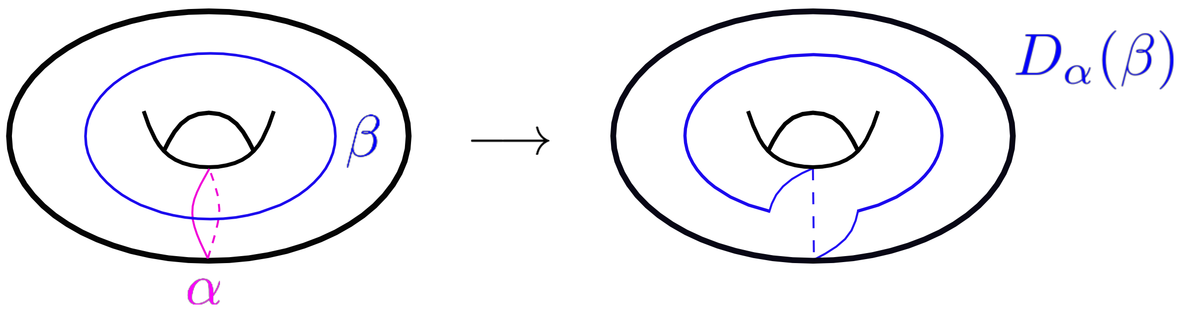

Pure mapping class groups

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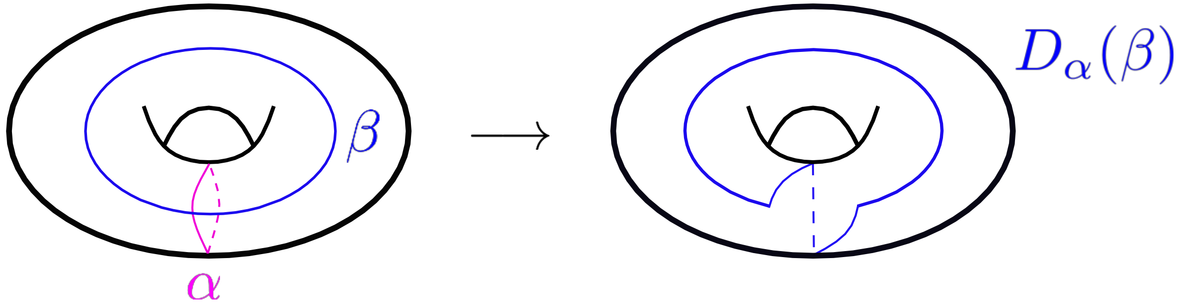
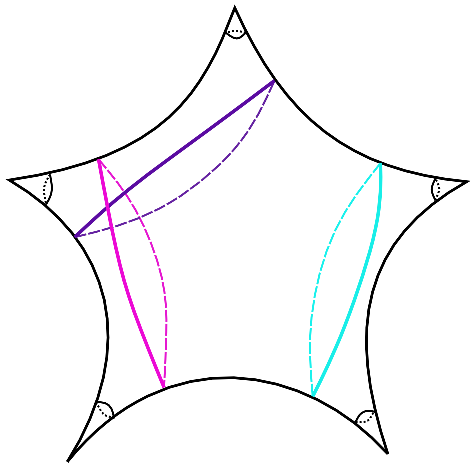
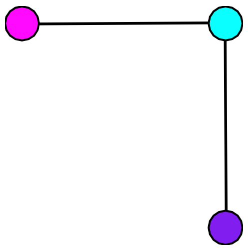


- Choice of generating set $\rightsquigarrow$ Cayley graph

Curve Graph

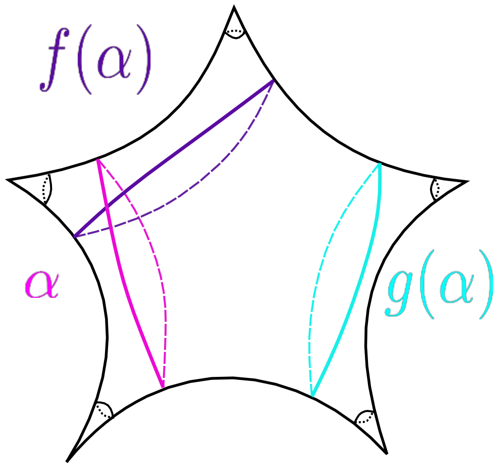
- Vertices = simple closed curves
- Edges = two curves are disjoint

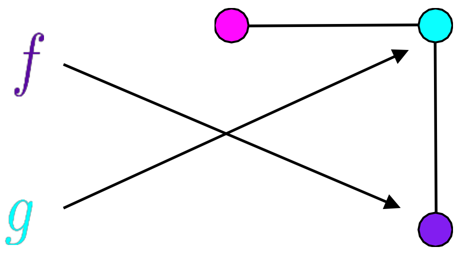




Shadow Map

$$\Upsilon : \mathrm{PMCG}(S_{0,5}) \to \mathscr{C}(S)$$
$$f \mapsto f(\alpha)$$





Theorem (Rafi - V.)

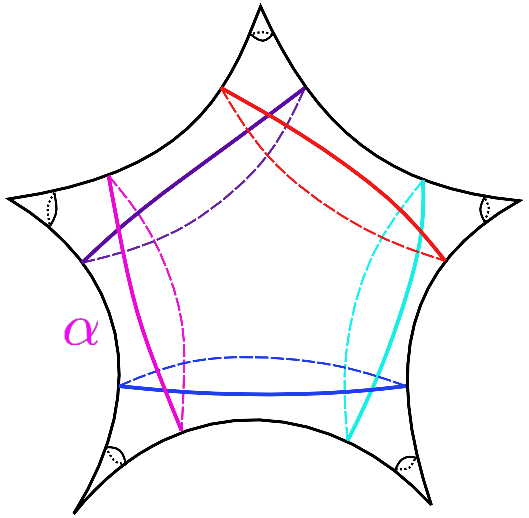
There is an $n \gg 1$, and a generating set $\mathcal{S}_n$ of PMCG$(S_{0,5})$ so that for every $K, C > 0$ there exists an $\mathcal{S}_n$ geodesic whose shadow to the curve graph $\mathscr{C}(S)$ is not a reparametrized (K, C)–quasi-geodesic.

Theorem (Masur - Minsky)

Every pair of elements in the mapping class group can be connected by a quasi-geodesic whose shadow to the curve graph can be reparametrized to a uniform quasi-geodesic.

Set-up

- Construct a generating set, $\mathcal{S}_n$



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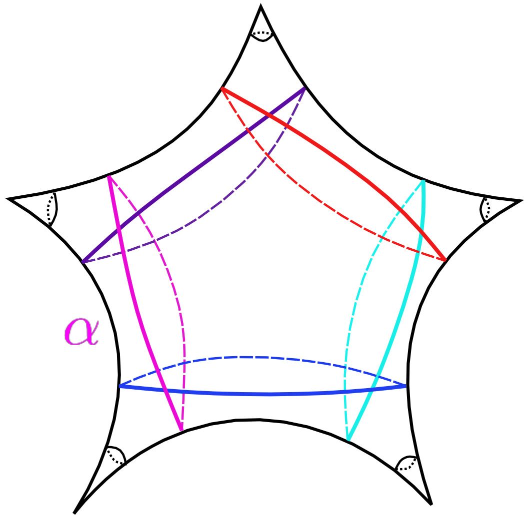
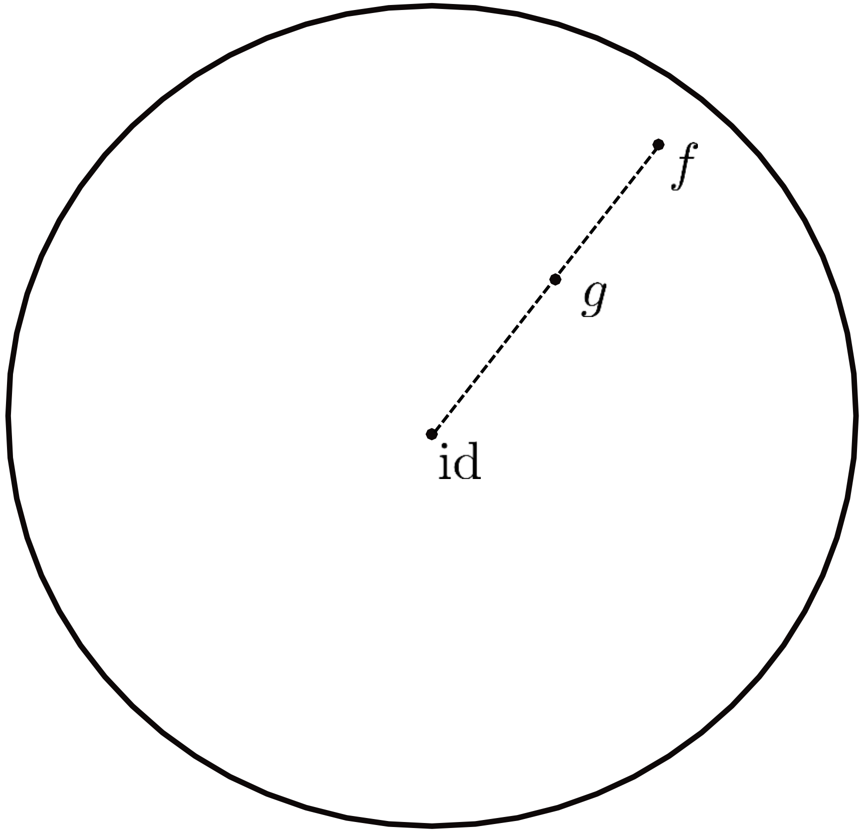


Proof Idea

$\mathrm{PMCG}(S_{0,5})$



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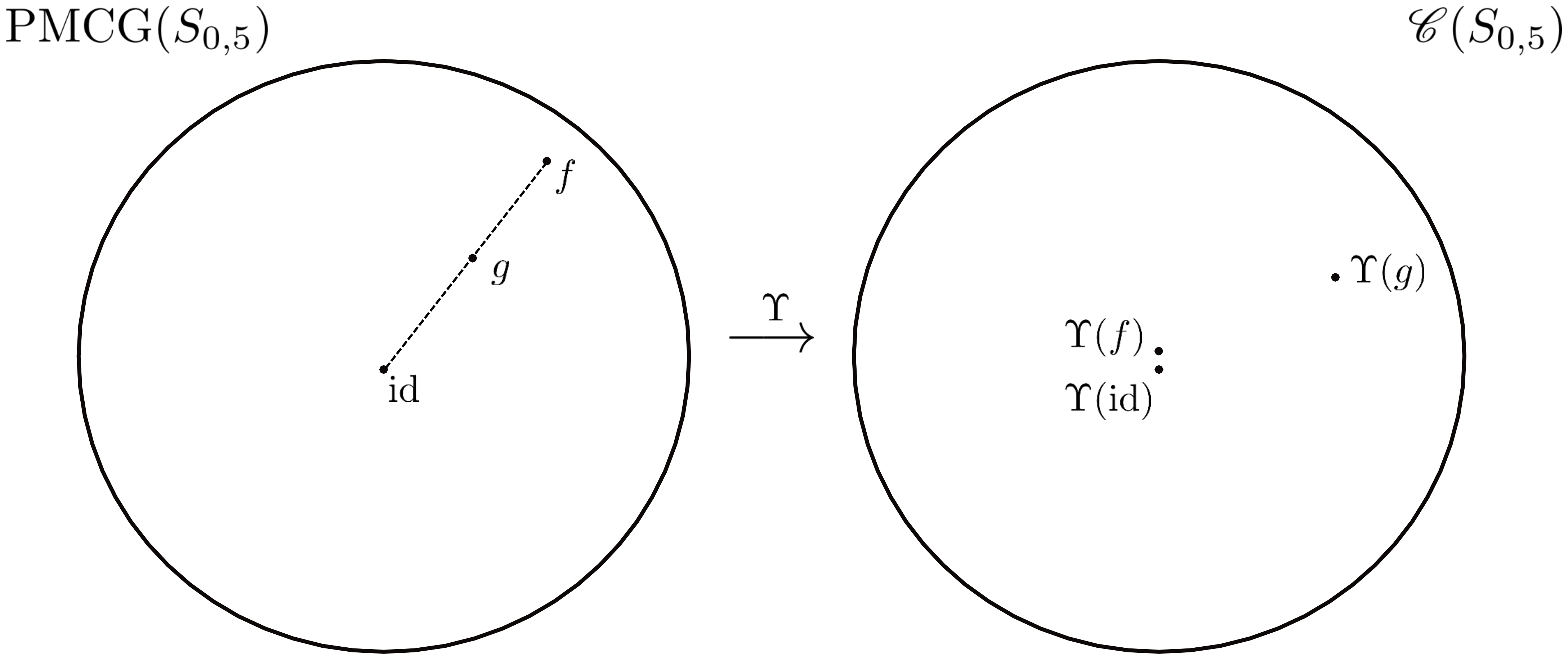
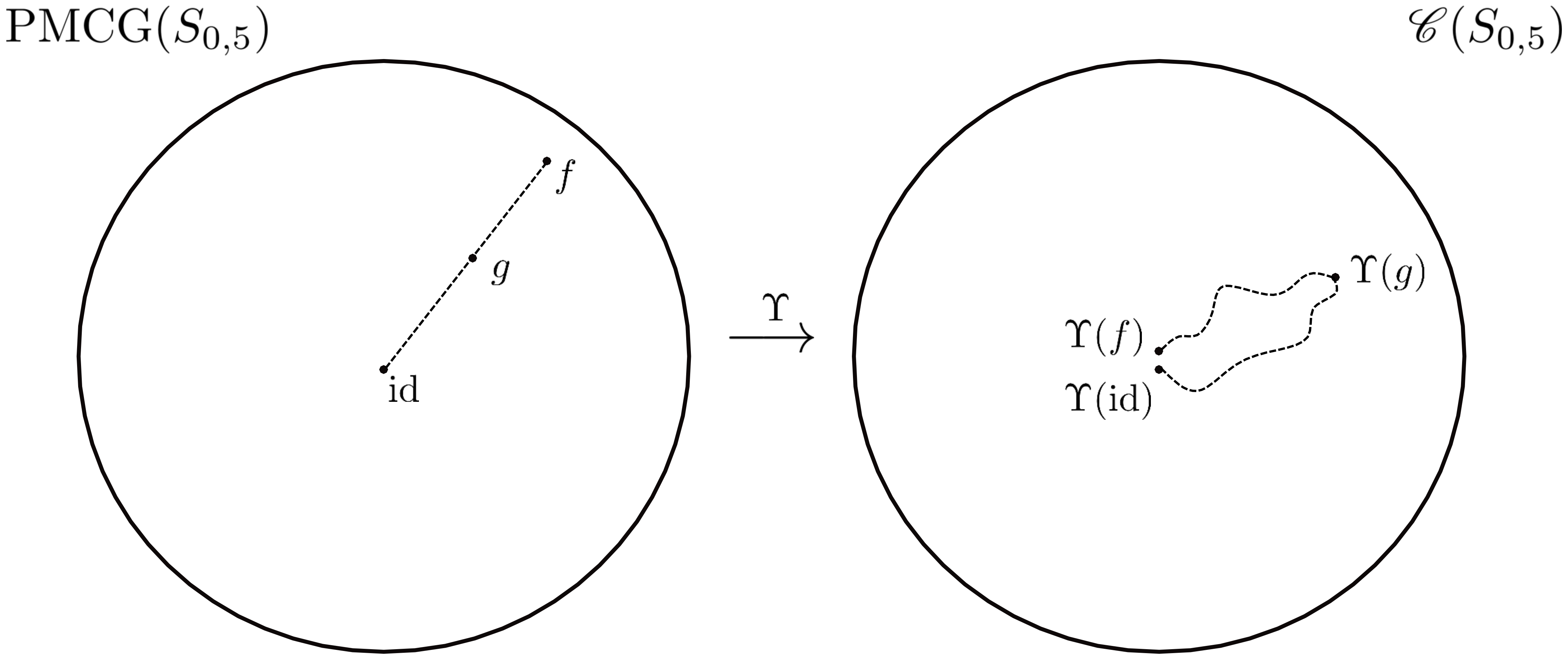


PMCG($S_{0,5}$)
$\mathscr{C}(S_{0,5})$
f
g
id
Υ
$\Upsilon(g)$
$\Upsilon(f)$
$\Upsilon(\mathrm{id})$

Proof Idea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Thank You