Prescribed virtual torsion in the homology of 3-manifolds

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based on joint work with Daniel Groves

Introduction

Let M be a compact irreducible 3-manifold with empty or toroidal boundary which is not a graph manifold.

Geometrization thm

M can be cut along tori into pieces which are either hyperbolic or Seifert-fibered.

not a graph manifold = at least 1 hyperbolic piece.

Introduction

Goal: understand the homology of finite covers of M

* M virtually contains any prescribed torsion in homology

$$H_1(M, \mathbb{Z}) = \mathbb{Z}^{b_1} \oplus T_1$$

torsion
finite abelian group

$b_1 = 1^{st}$ Betti number

Motivation and History

Lück approximation thm — combined Lott-Lück :

$\bigcap \Gamma_i = \{1\}$

If $\longrightarrow \overset{\Gamma_i}{M_i} \longrightarrow \overset{\Gamma_2}{M_2} \longrightarrow \overset{\Gamma_1}{M_1} \longrightarrow \overset{\Gamma}{M}$ tower of cofinal regular covers of M

$$\lim_{i\to\infty} \frac{b_1(M_i)}{[\Gamma : \Gamma_i]} = 0 .$$

Motivation and History

Torsion growth conjecture (Bergeron–Venkatesh, Lück, Le)

There is a cofinal tower regular covers of M s.t.

$$\limsup_{i\to\infty} \frac{\ln|T_1(M_i)|}{[\Gamma:\Gamma_i]} = \frac{\mathrm{vol}(M)}{6\pi}$$

vol(M) = Σ of volumes of the hyp. pieces

Le: for any such tower

$$\limsup_{i\to\infty} \frac{\ln|T_1(M_i)|}{[\Gamma:\Gamma_i]} \leq \frac{\mathrm{vol}(M)}{6\pi}$$

Motivation and History

A weaker question:

Given M, does there exist a finite cover $\tilde{M} \to M$ s.t. $T_1(\tilde{M}) \neq 0$?

- Sun 2015: true for closed hyperbolic 3-manifolds
- indep. Friedl-Hermann, Liu $\}$ true for M as before

Motivation and History

Theorem (Sun 2015)

Let N be a closed hyperbolic 3-manifold.

Let A be any finite abelian group.

Then there is a finite cover $\tilde{N} \to N$ such that

A is a direct summand in $H_1(\tilde{N}, \mathbb{Z})$

this is what I call "prescribed torsion"

Statement

Theorem (C-Groves)

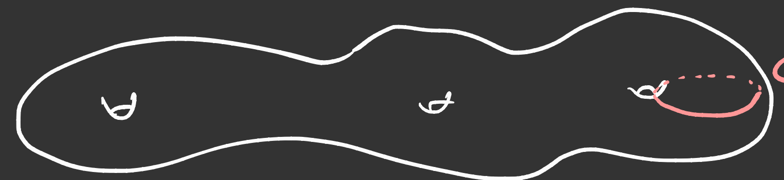
Let M be a compact irreducible 3-manifold with empty or toroidal boundary which is not a graph manifold.

Let A be any finite abelian group.

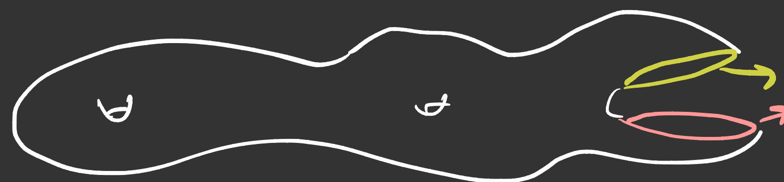
There is a $\tilde{M} \to M$ finite cover such that A is a direct summand in $H_1(\tilde{M}; \mathbb{Z})$.

Ideas

Surface Σ_g



cut along c

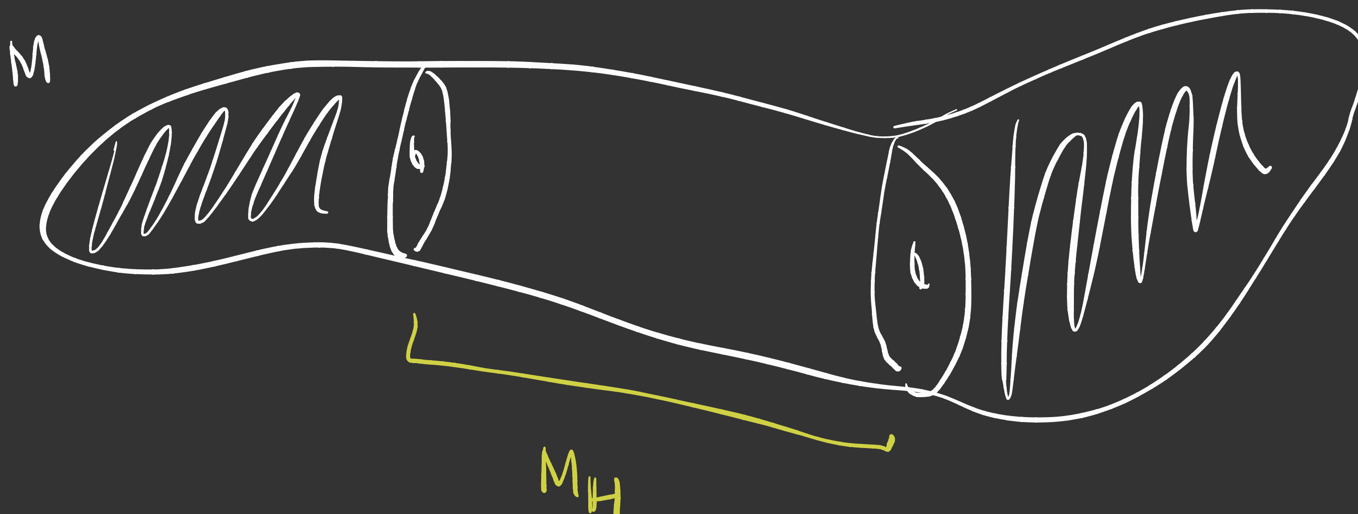


quotient by $\frac{2\pi}{n}$ rotation

X_n

Exercise: $H_1(X_n; \mathbb{Z}) = \mathbb{Z}^{2g-1} \oplus \mathbb{Z}/n\mathbb{Z}$.

Ideas



Ideas

- use the Kahn-Wright construction of surfaces to build a 2-complex $X_n \looparrowright M_{\mathbb{H}}$ such that
  - X_n stays far away from the cusps
  - immersion is π_1-injective
  - image of $\pi_1(X_n)$ be a quasi-convex subgp in $\pi_1(M_{\mathbb{H}})$

  *

- appeal to the virtually special thms (Agol, Wise, Przytycki-Wise) to find a finite index subgp $H \leq \pi_1(M)$ such that H retracts to $\pi_1(X_n)$.
- standard computation shows that
  $H_1(\tilde{M}, \mathbb{Z}) = H_1(X_n, \mathbb{Z}) \oplus \ker(r_*)$

  corresponding to H

Thank you!