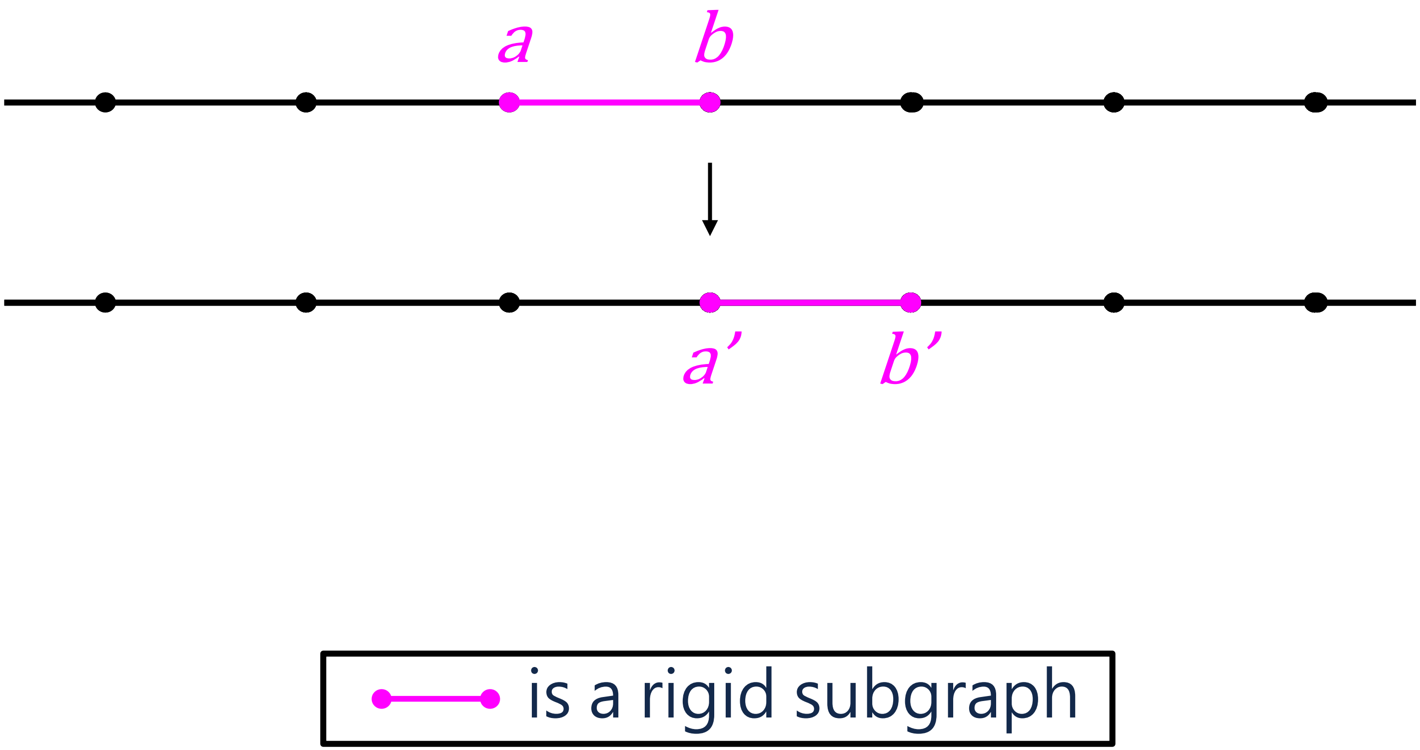
Finite Rigid Sets in Flip Graphs

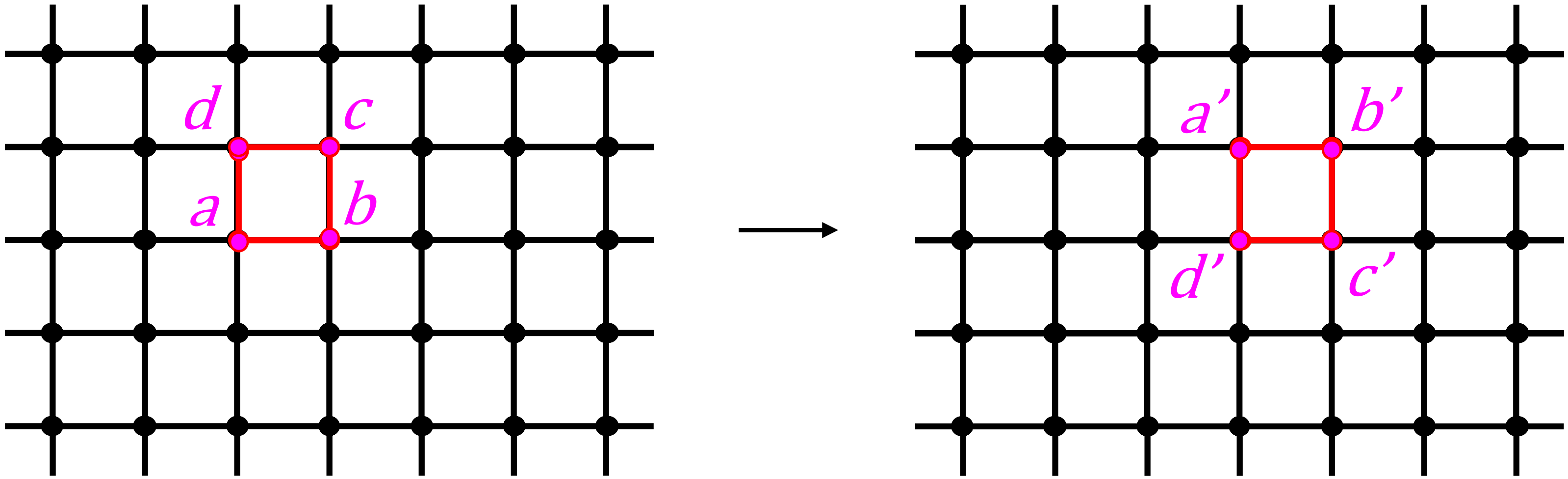
Emily Shinkle

ILLINOIS

Rigid Subgraphs



Rigid Subgraphs



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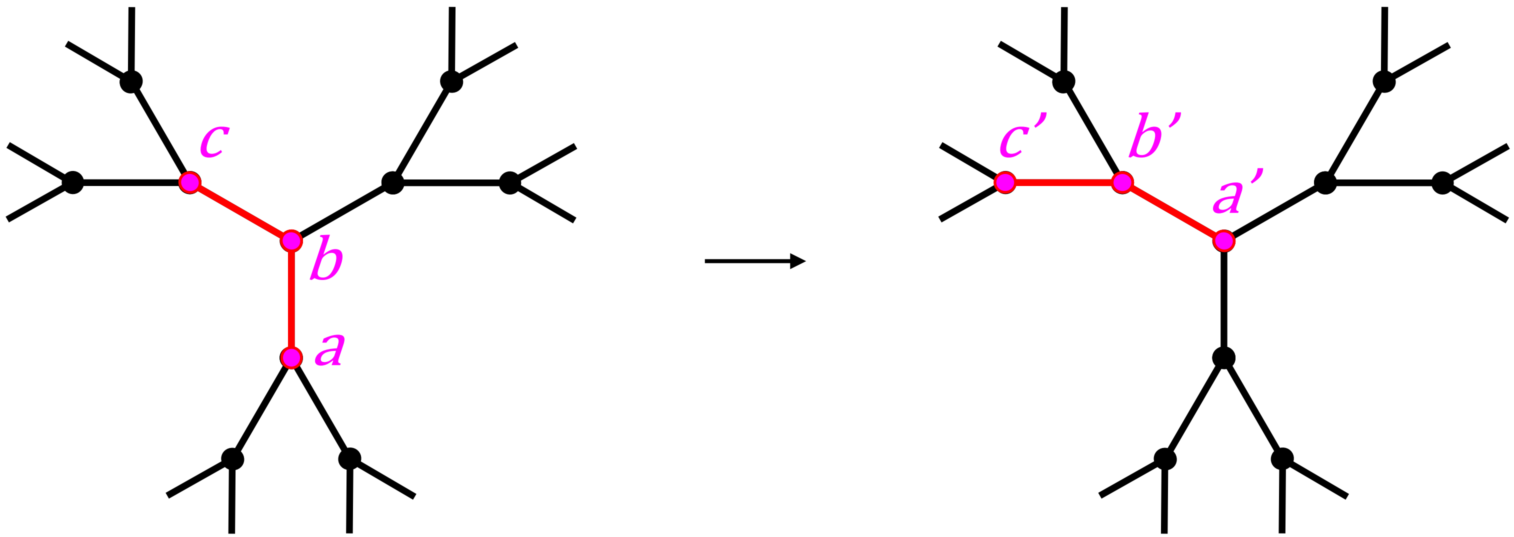
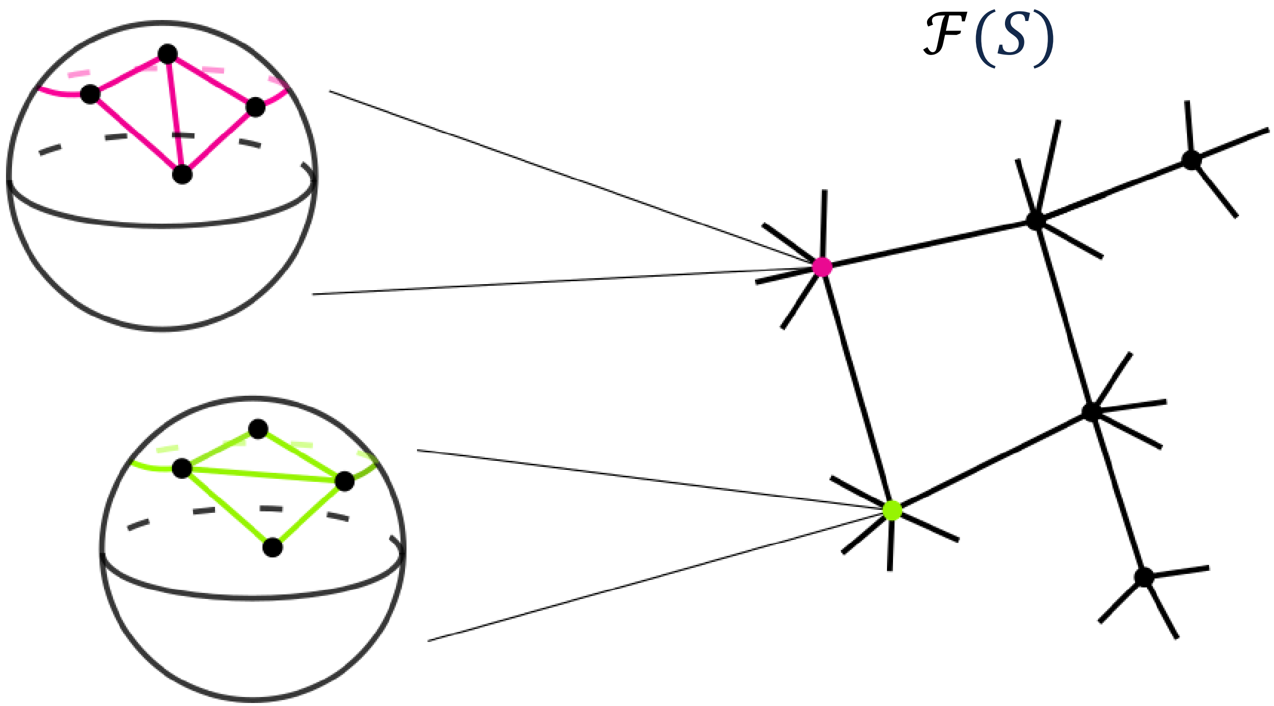


No finite rigid subgraphs

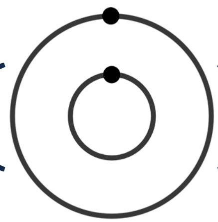
The Flip Graph

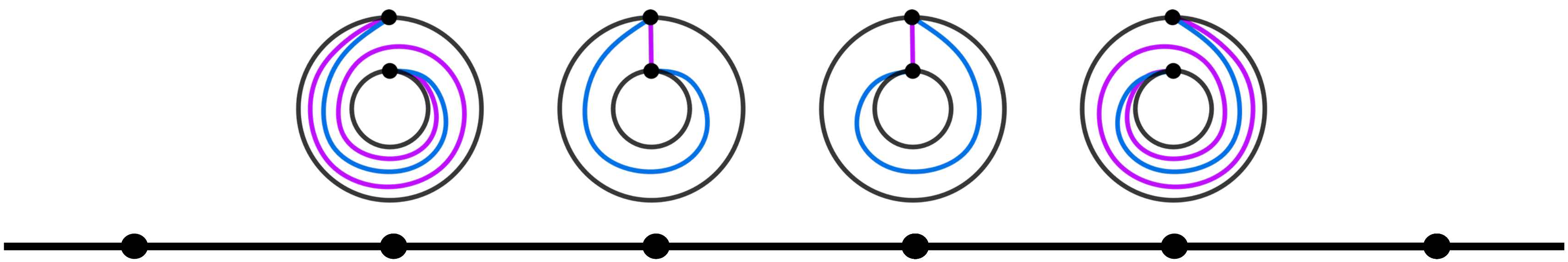
The Flip Graph $\mathcal{F}(S)$

vertices $\leftrightarrow$ triangulations on S
edges $\leftrightarrow$ flips

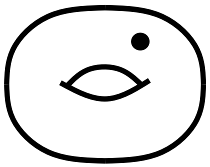


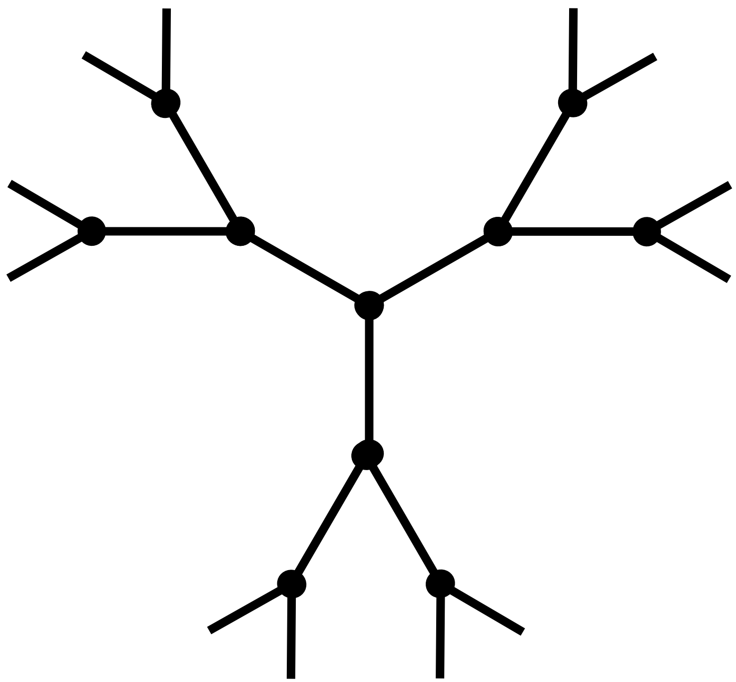
Examples

$\mathcal{F}$()



Examples

$\mathcal{F}($  $)$



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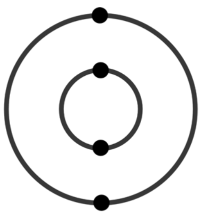
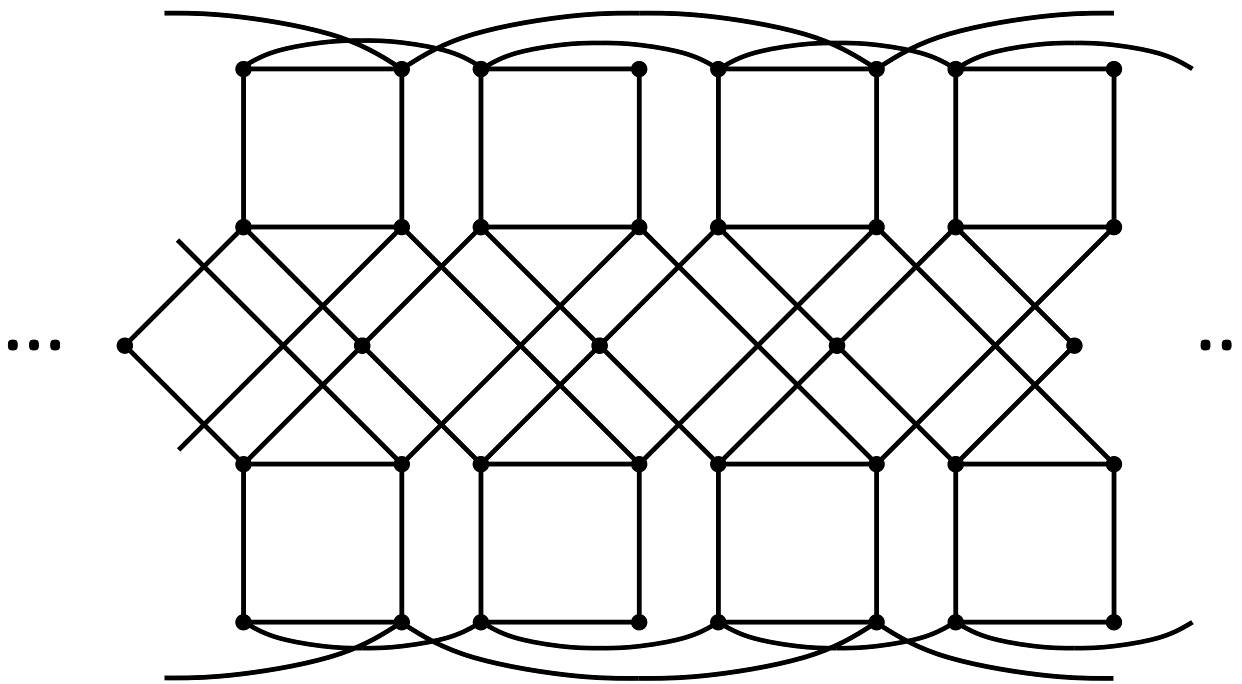
$\mathcal{F}($  $)$



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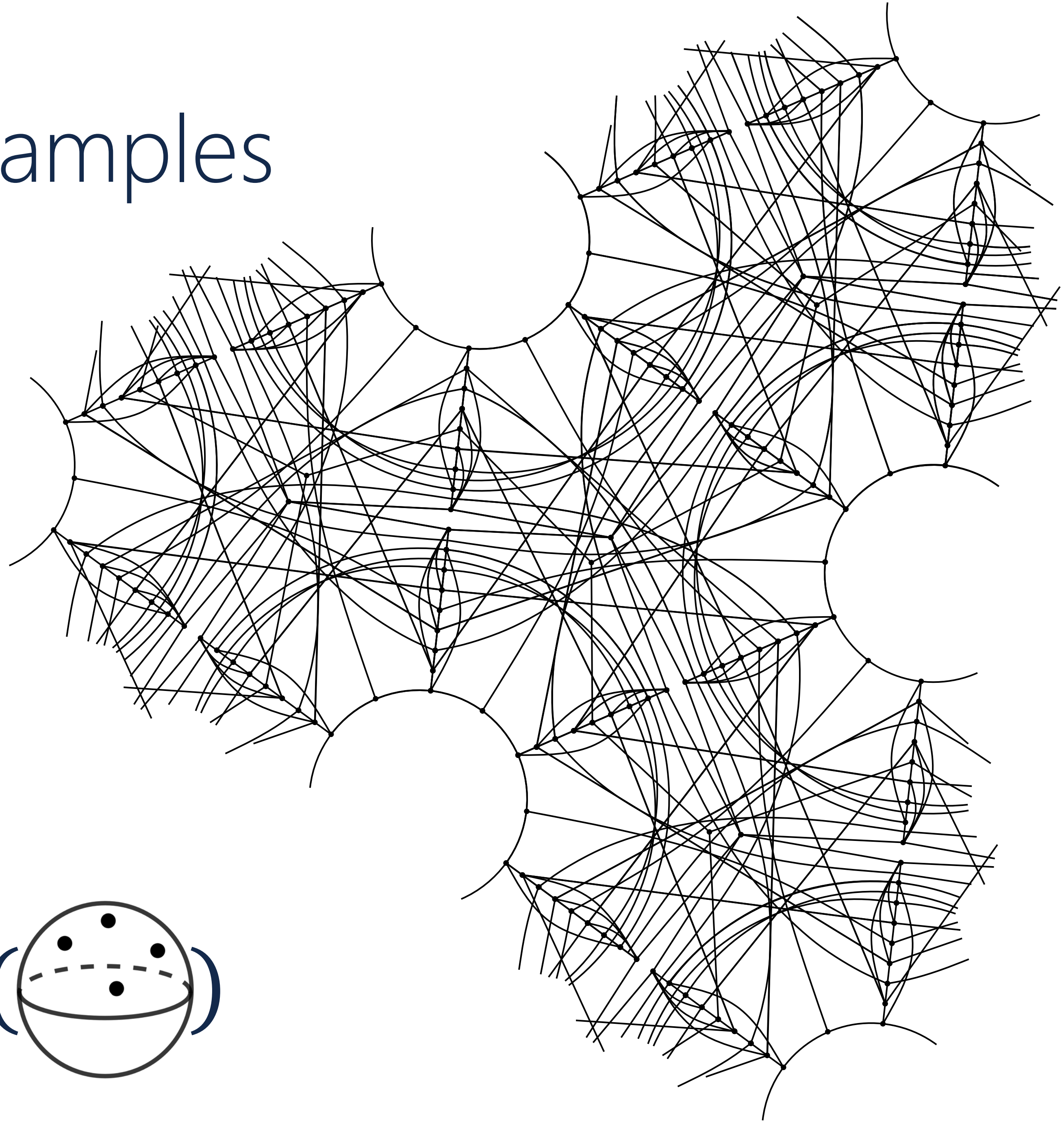
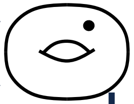


$\mathcal{F}($ $)$

Finite Rigidity of $\mathcal{F}(S)$

Theorem (S., 2020)

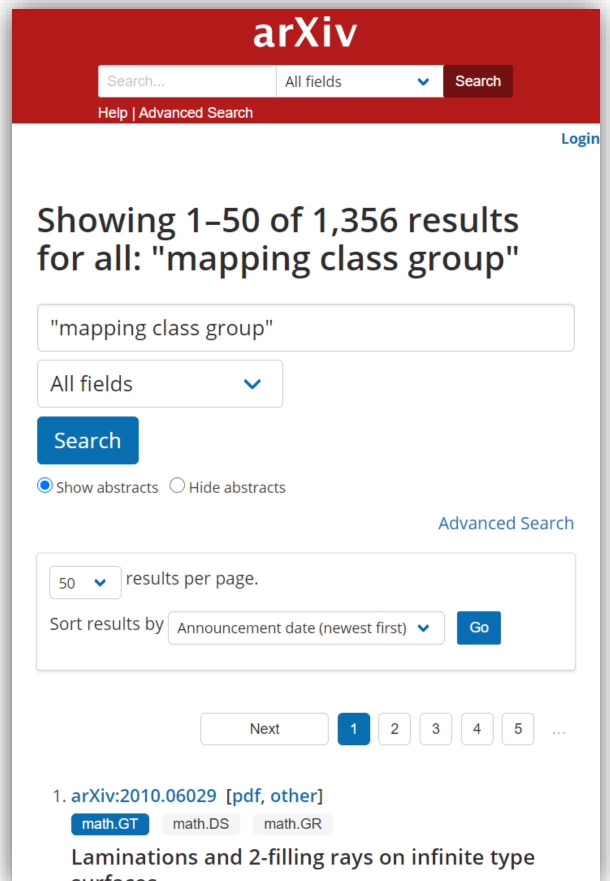
Besides $\mathcal{F}($$)$, every flip graph has a finite rigid subgraph.

Why do we care about these graphs?

(Extended) Mapping Class Group $\mathrm{Mod}^{\pm}(S)$

$$\left\{ \begin{array}{c} \text{homeomorphisms} \\ S \to S \\ \text{up to isotopy} \end{array} \right\}$$

- "Symmetries" of surface
- Algebraic invariant of surface
- Modular group of Teichmüller space



$$\mathrm{Mod}^{\pm}(S) \curvearrowright \mathcal{F}(S)$$

Theorem (Korkmaz-Papadopoulus, 2012; Aramayona-Koberda-Parlier, 2015; S., 2020)

Besides a few exceptional surfaces,

$$\left\{ \begin{array}{c} \text{automorphisms} \\ \mathcal{F}(S) \to \mathcal{F}(S) \end{array} \right\} \Longleftrightarrow \left\{ \begin{array}{c} \text{homeomorphisms} \\ S \to S \\ \text{up to isotopy} \end{array} \right\}$$

$$\left\{ \begin{array}{c} \text{inj. simplicial maps} \\ \mathcal{X} \to \mathcal{F}(S) \end{array} \right\}$$

$$\Longleftrightarrow$$

$$\left\{ \begin{array}{c} \text{automorphisms} \\ \mathcal{F}(S) \to \mathcal{F}(S) \end{array} \right\}$$

$$\Longleftrightarrow$$

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Proof Ideas

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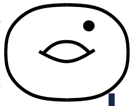
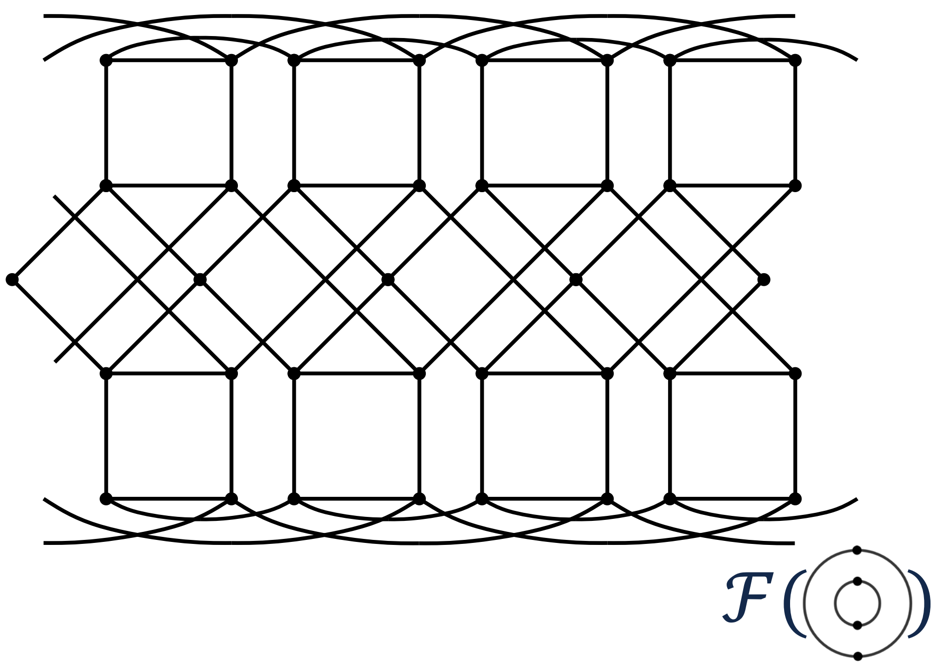
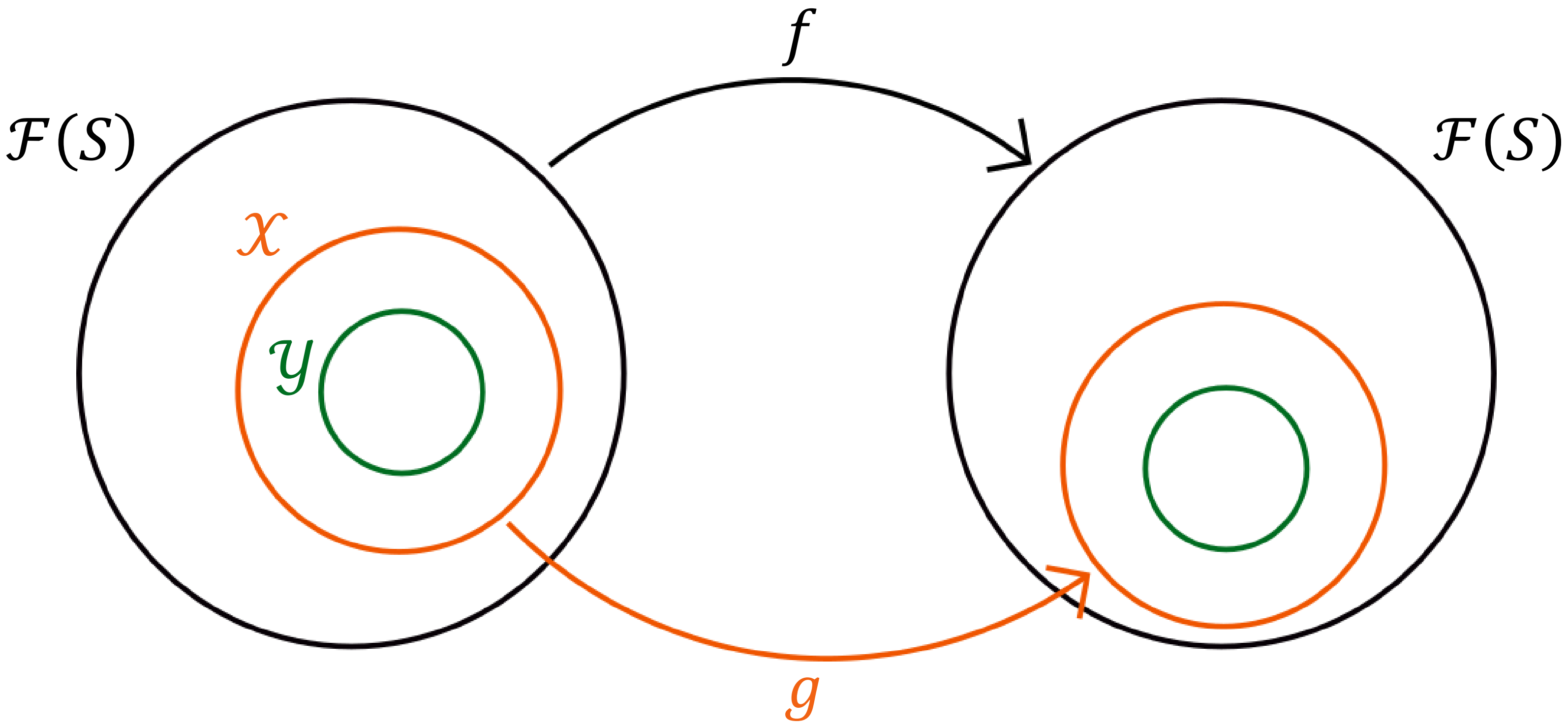
Besides $\mathcal{F}($$)$, every flip graph has a finite rigid subgraph.

Helpful properties

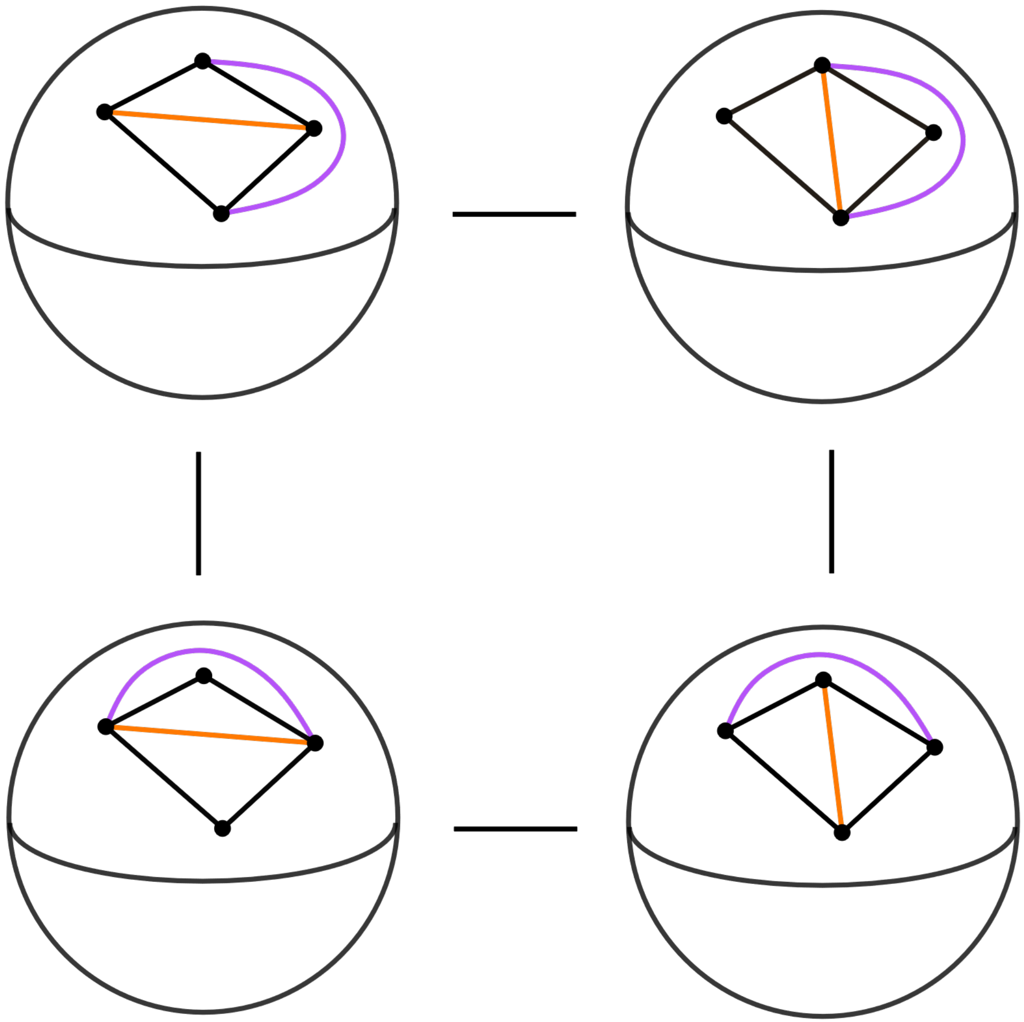
- Connected
- Locally finite
- Finitely many automorphism classes of vertices



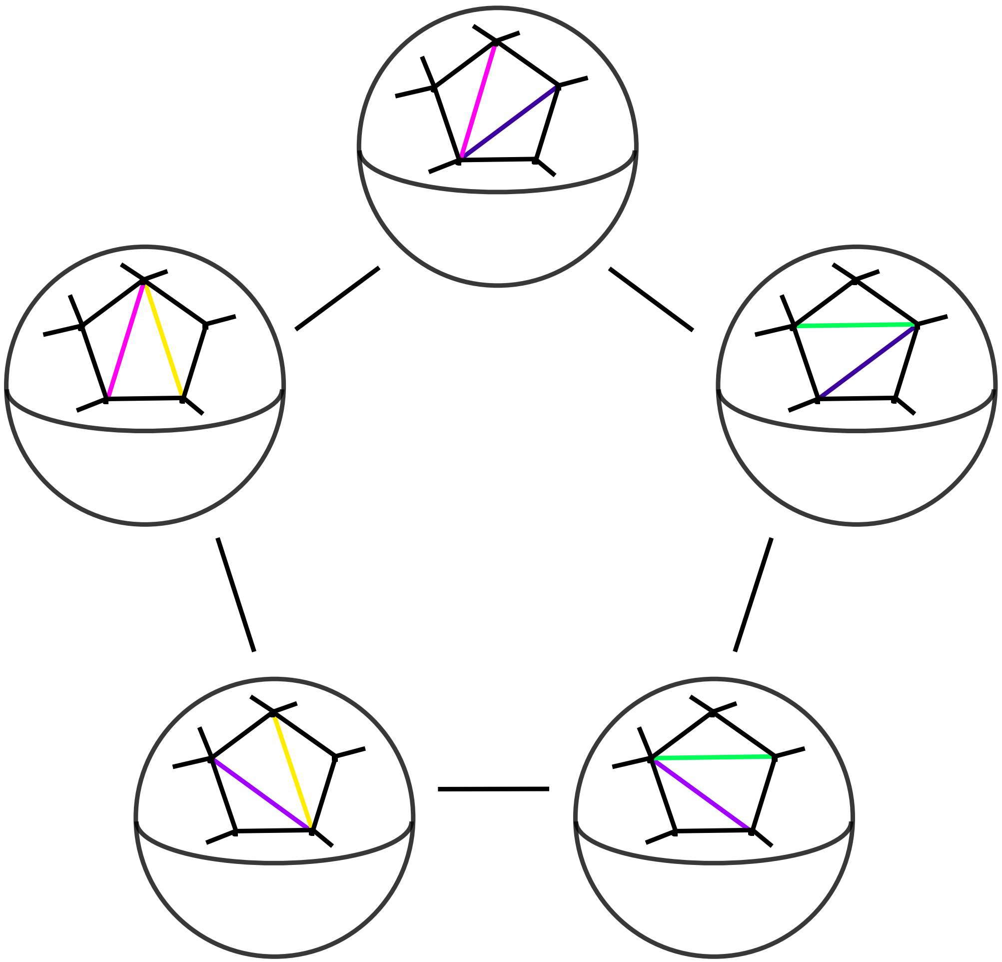
Almost extension



Four-cycle



Five-cycle



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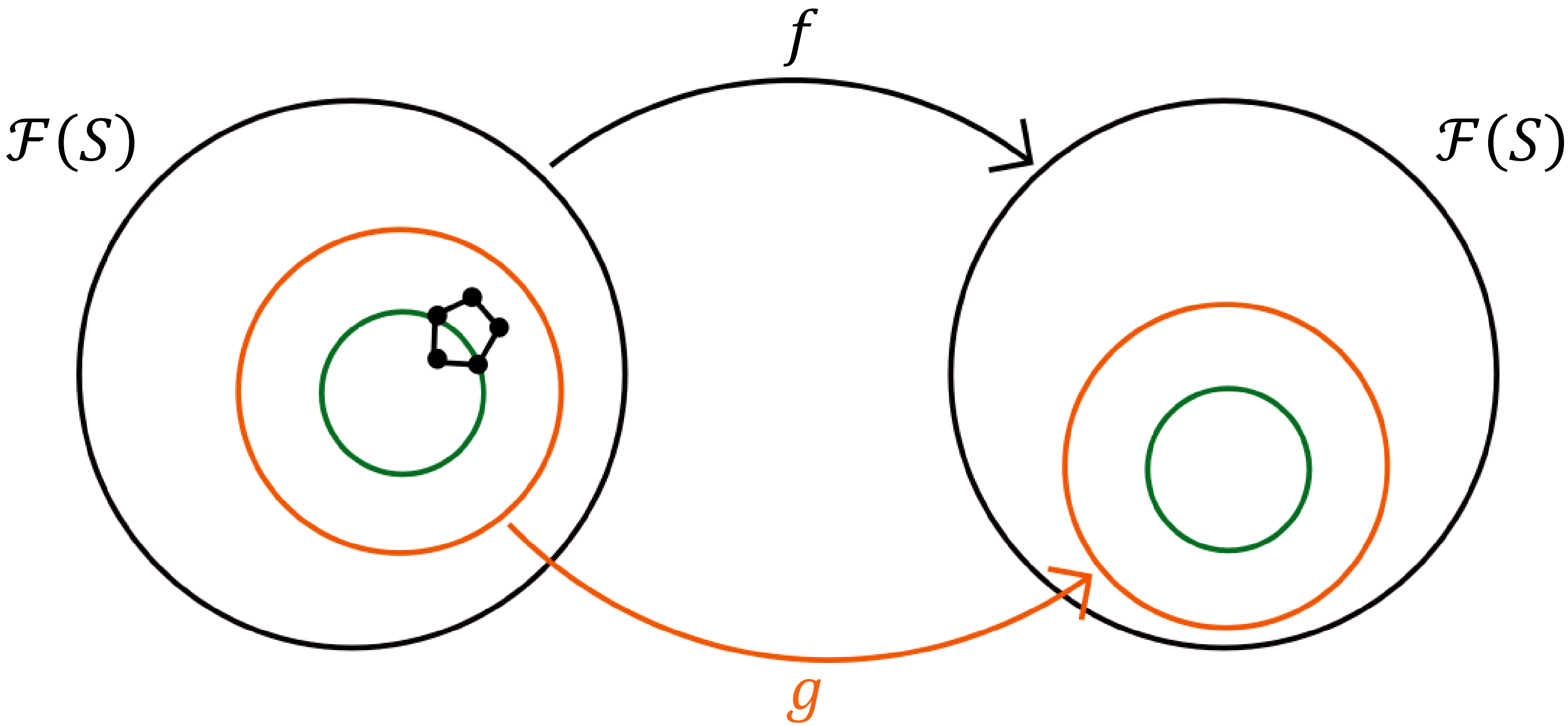

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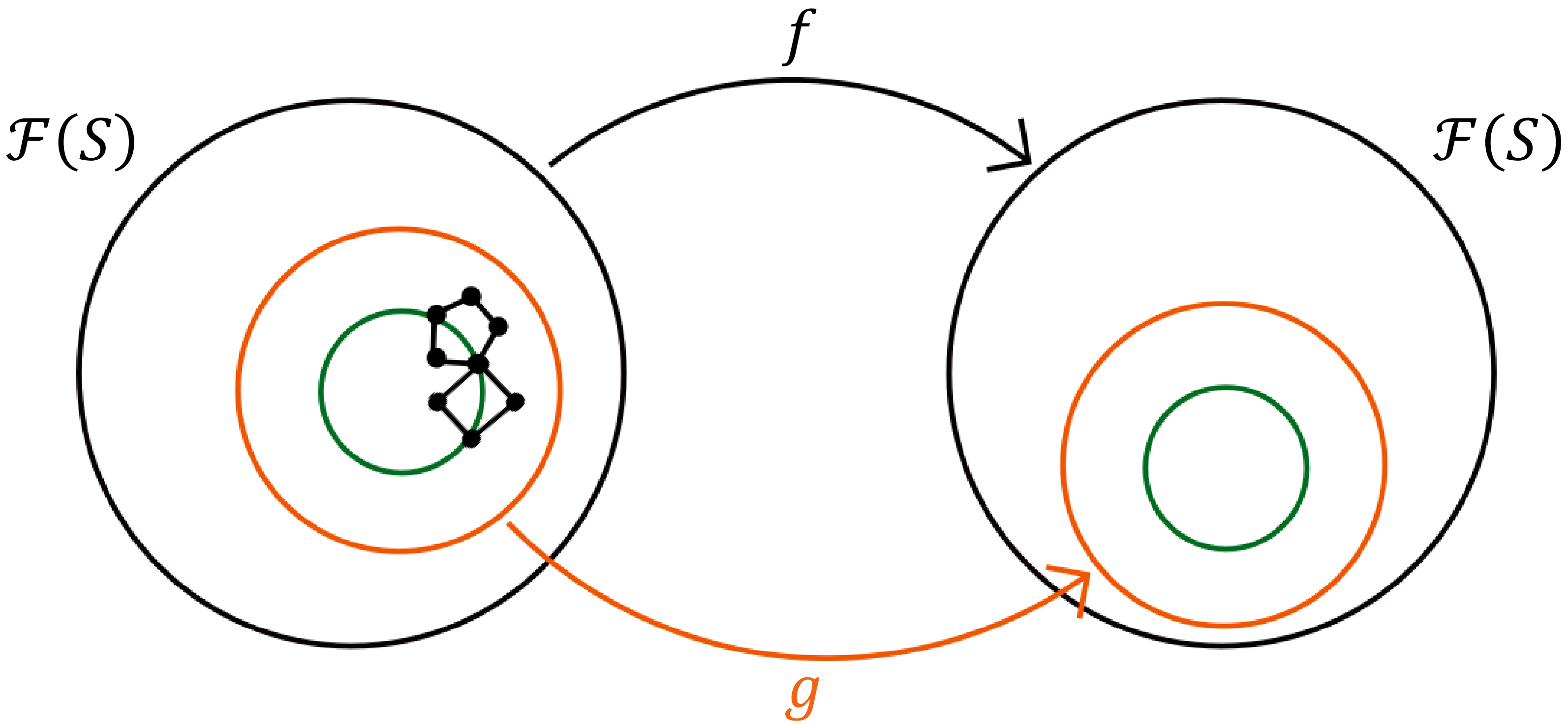

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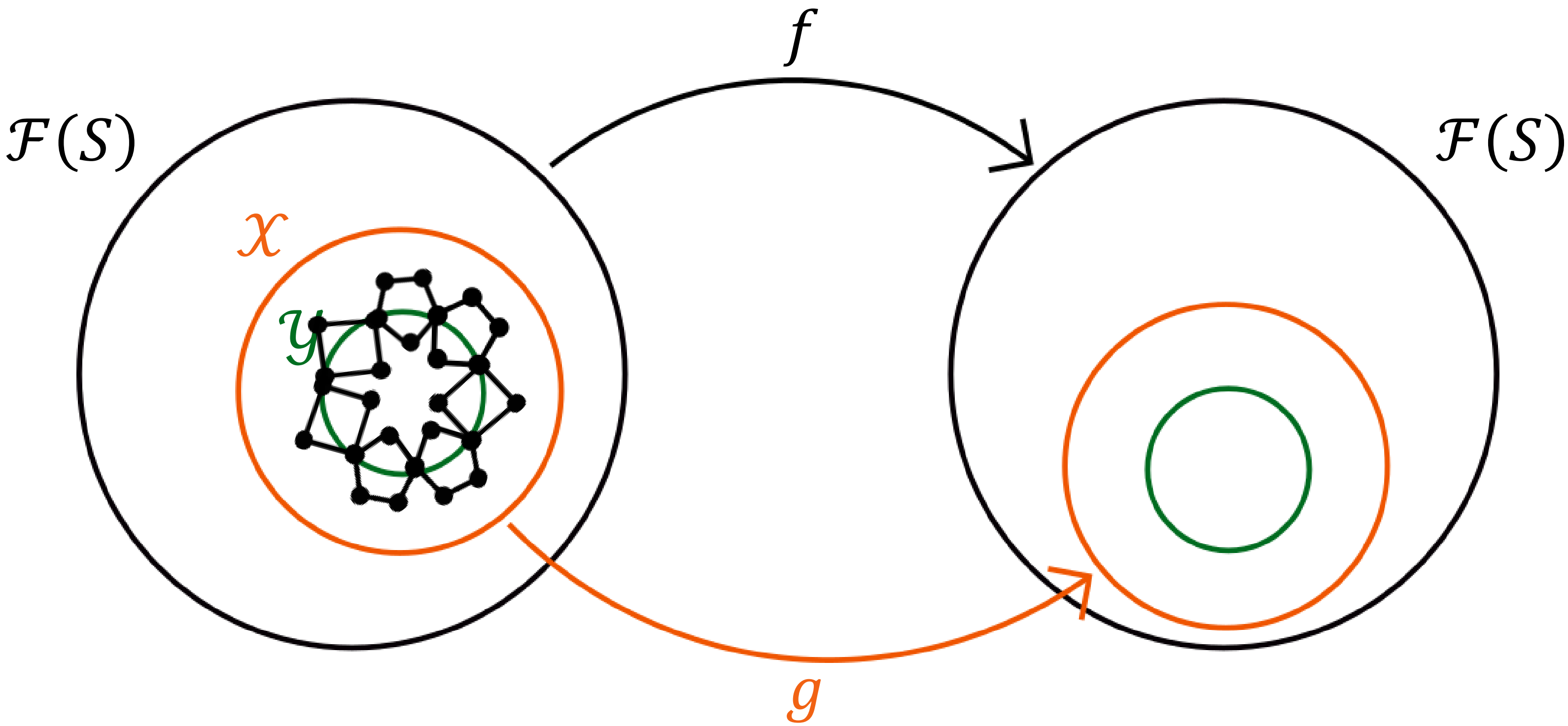
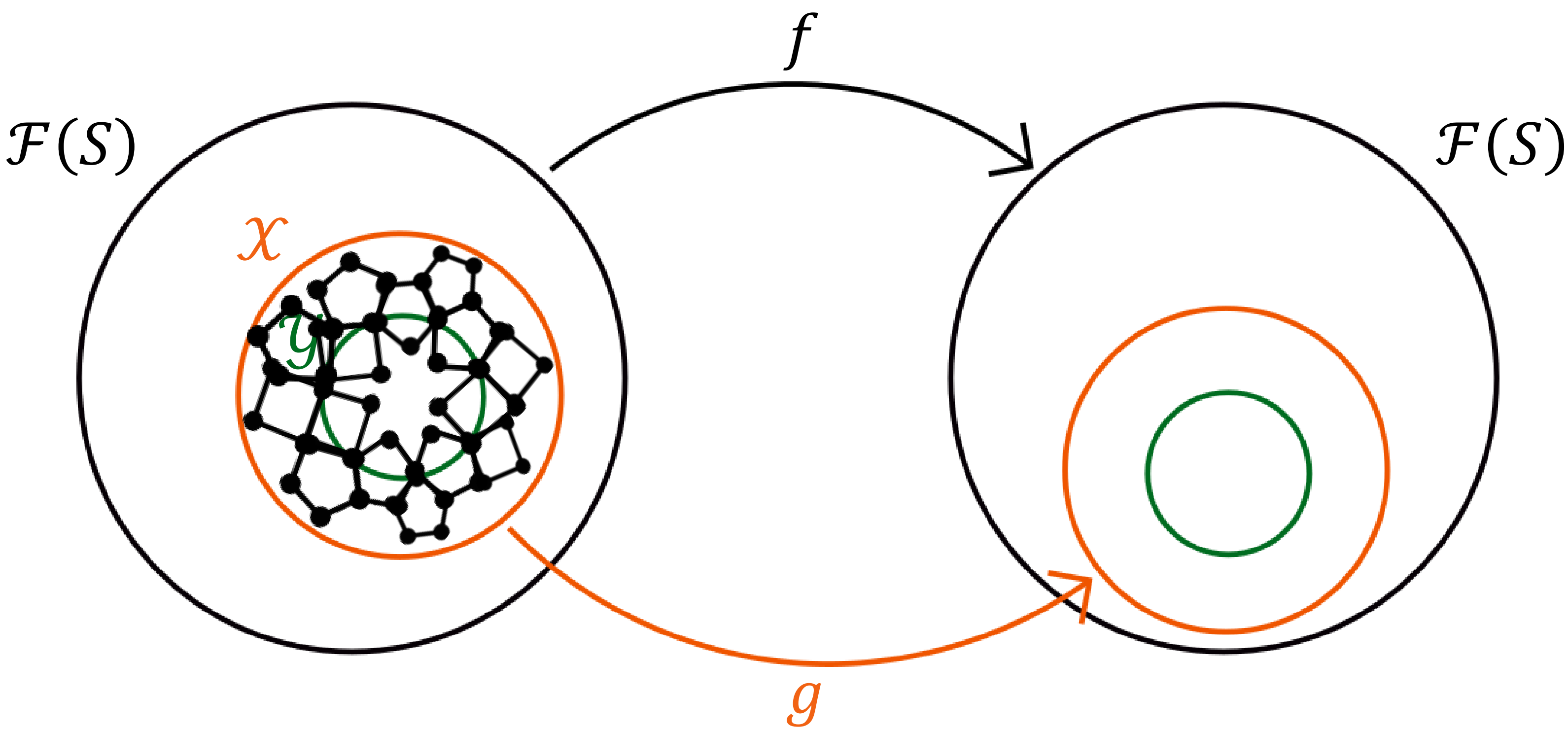


Extension!



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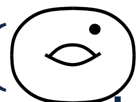
Besides $\mathcal{F}($$)$, every flip graph has a finite rigid subgraph.

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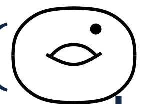
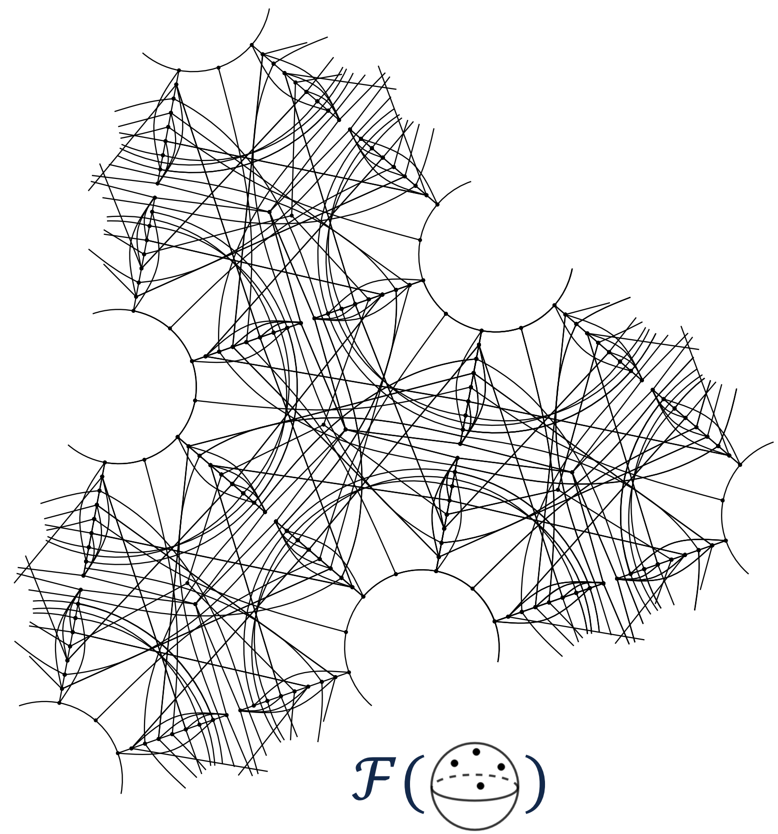
Besides $\mathcal{F}($$)$, every flip graph has a finite rigid subgraph.

Thank you for your time!



Symplectic fillings of lens spaces

Agniva Roy
Joint work with John Etnyre

Georgia Tech

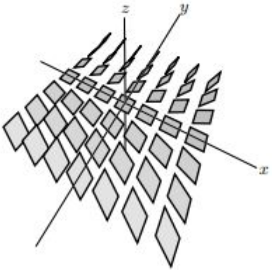
Tech Topology - December 2020

Contact structures and contact manifolds

M^3

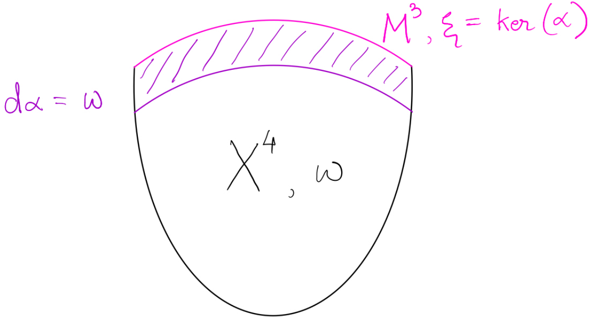
α 1-form such that $\alpha \wedge (d\alpha) > 0$
$\xi = ker(\alpha)$ 2-plane distribution

Example: $\mathbb{R}^3, \xi_{std} = ker(dz - ydx)$



$\mathbb{R}^3, \xi_{std}$

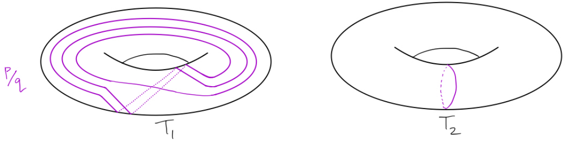
Symplectic fillings



(X, w) symplectic filling of (M, ξ)

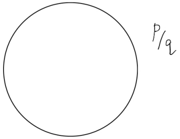
Question: Given (M, ξ), can you classify all of its symplectic fillings?

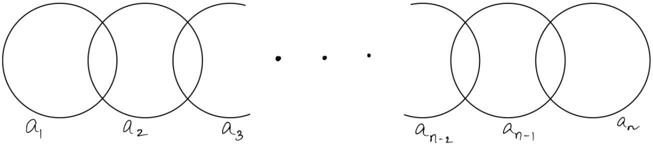
Lens spaces



Gluing two solid tori together

Lens spaces





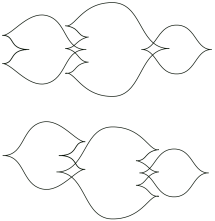
Surgery pictures, $-\frac{p}{q} = a_1 - \cfrac{1}{a_2 - \cfrac{1}{\cdots - \cfrac{1}{a_n}}}$

Tight structures on lens spaces - Giroux, Honda, 2000

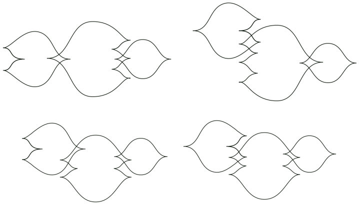
Example: $L(13,8)$

$$-\frac{13}{8} = -3 - \frac{1}{-4 - \frac{1}{-2}}$$

Universally tight:



Virtually overtwisted:



Symplectic fillings of tight lens spaces (upto diffeomorphism)

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- Fillings of some classes of virtually overtwisted structures were classified by Plamenevskaya-van Horn Morris (2012), Kaloti (2015), Fossati(2018).
- Etnyre-R. and independently Christian-Li (2020) classified all minimal symplectic fillings of all virtually overtwisted lens spaces.

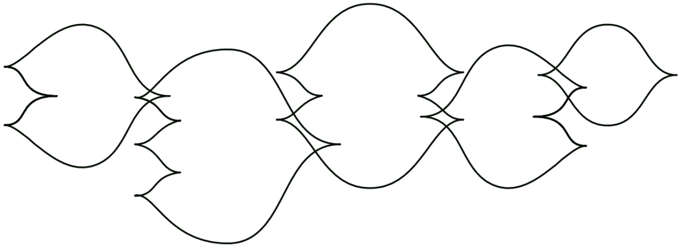
Main result

Theorem (Etnyre-R., independently Christian-Li): The minimal symplectic fillings of $(L(p,q), \xi_{vo})$, as smooth manifolds, are a subset of the minimal symplectic fillings of $(L(p,q), \xi_{ut})$.

Technology: Menke's(2018) result on symplectic fillings of contact 3-manifolds containing **mixed tori**.

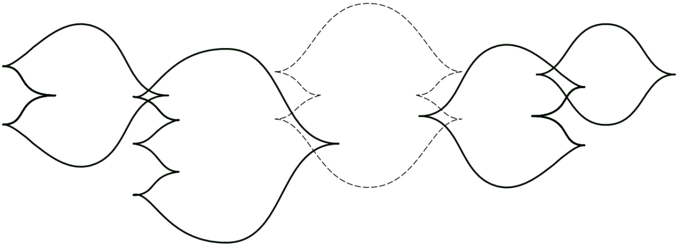
Constructing Stein fillings - algorithm

Example: Virtually overtwisted structure



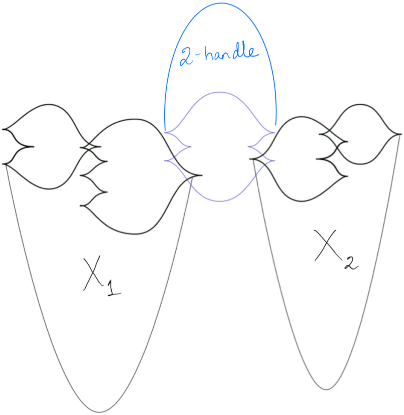
Constructing minimal fillings - algorithm

Step 1: Remove knots to get a union of consistent chains



Constructing minimal fillings - algorithm

Step 2: Take fillings of consistent chains, add 2-handles along the knots that were removed



Consequence:

Corollary 1: The minimal filling of a lens space with maximal b_2 is unique and given by the plumbing.

Corollary 2: There exist no nontrivial Stein cobordisms from $(L(p,q),\xi_1)$ to $(L(p,q),\xi_2)$, where ξ_1 and ξ_2 are tight.

More on symplectic cobordisms between tight lens spaces

Corollary 2': If there exists a Stein cobordism from $(L(p,q),\xi)$ to $(L(p',q'),\xi')$, then $l(p/q) \leq l(p'/q')$. In case of equality, it must be trivial.

$$-\frac{p}{q} = a_1 - \cfrac{1}{a_2 - \cfrac{1}{\dots - \cfrac{1}{a_n}}} \implies l(p/q) := n$$

Question: Does there exist a Stein cobordism from a virtually overtwisted lens space to an universally tight lens space?

Contact lens space realisation

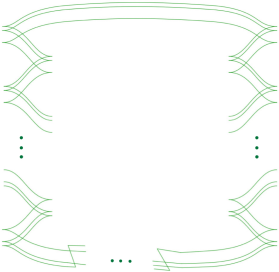
Question: Which tight lens spaces can be obtained by Legendrian surgery on a single knot in (S^3, ξ_{std})?

Contact lens space realisation

Question: Which tight lens spaces can be obtained by Legendrian surgery on a single knot in (S^3, ξ_{std})?

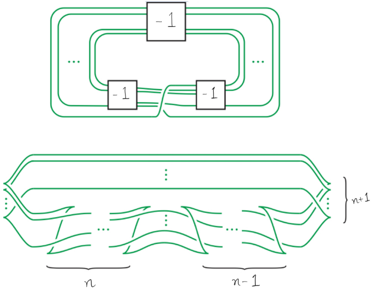
Conjecture: Only the following families:

$L(nm+1, m^2)$



$(n, -m)$ torus knot

$L(3n^2+3n+1, 3n+1)$



A type of Berge knot

Vector fields, mapping class groups, and holomorphic 1-forms

Aaron Calderon

Yale University

(joint work w/ Nick Salter)

Framed surfaces

Framing ≈ nonvanishing vector field

Signature = $(k_1, \ldots, k_n)$

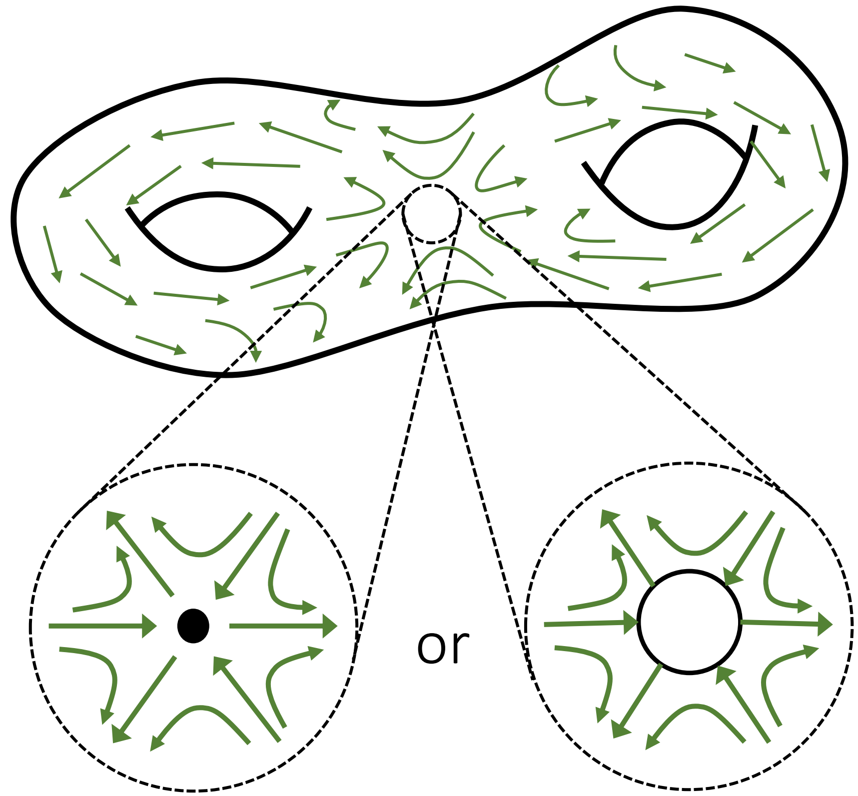
$k_i = -$ (index of i^{th} boundary)

Poincaré–Hopf: $\Sigma\, k_i = 2g\text{-}2$

winding number functions:

wn(f): $\{\overrightarrow{\text{curves}}\} \longrightarrow \mathbb{Z}$

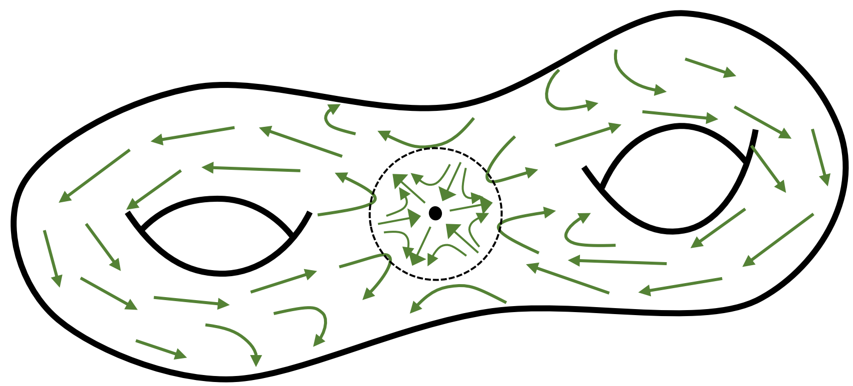
wn(F): $\{\overrightarrow{\text{curves}}\} \sqcup \{\overrightarrow{\text{arcs}}\} \longrightarrow \mathbb{Z}$



f: punctures

F: boundary

Framed surfaces



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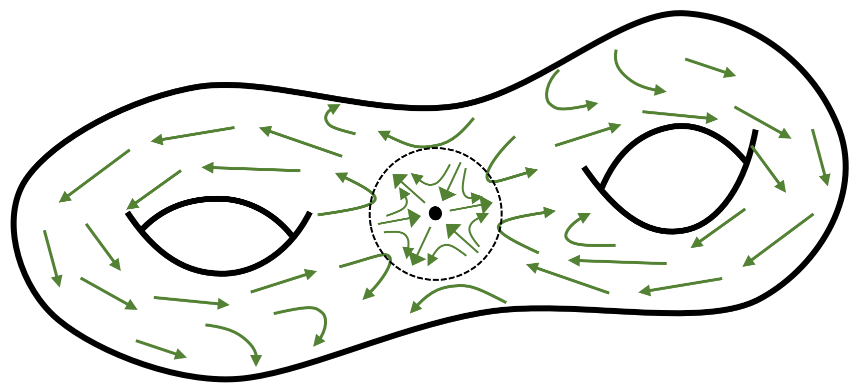
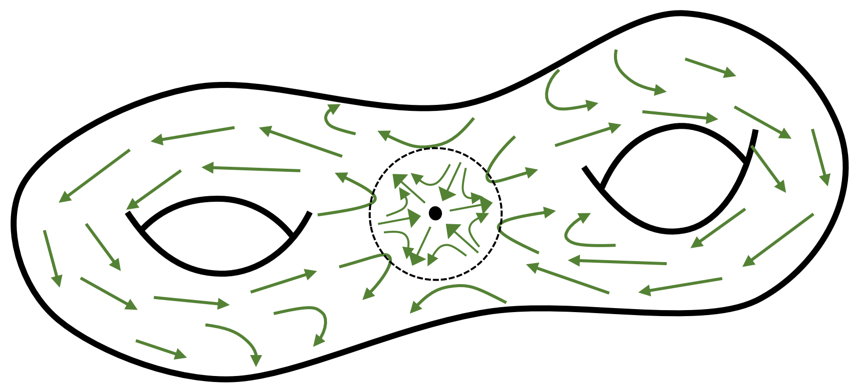


Framed MCGs: fMod & FMod
(punctures) (boundary)

stabilize f/F up to isotopy
⟺ preserve all winding #s

infinite index, not normal

Framed surfaces



Framed MCGs: fMod & FMod
(punctures) (boundary)

stabilize f/F up to isotopy
$\Longleftrightarrow$ preserve all winding #s

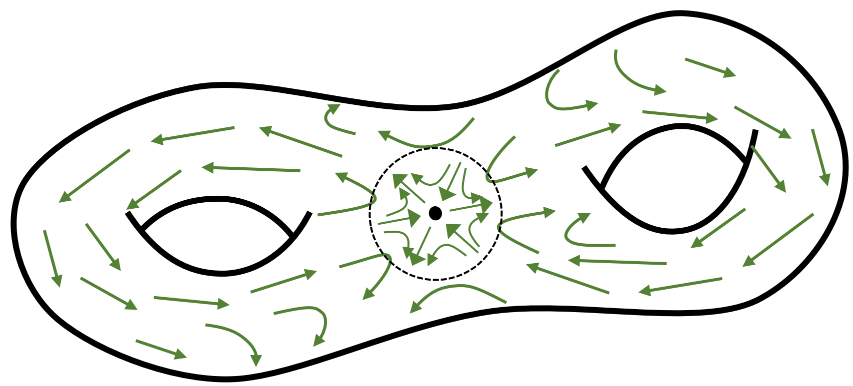
infinite index, not normal

Framed surfaces

<u>Twist-linearity:</u>
γ an arc or curve, c a curve.
$\mathrm{wn}(T_c\gamma) = \mathrm{wn}(\gamma) + \langle\gamma, c\rangle\, \mathrm{wn}(c)$

wn(c) = 0 "admissible"
⇝ $T_c \in$ fMod and FMod

c separating
⇝ $T_c \in$ fMod but *not* FMod



Framed MCGs: fMod & FMod
(punctures) (boundary)

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infinite index, not normal

Theorem: [C.–Salter, '20]

Let $\underline{k}$ be a partition of $2g - 2$ with $g \geq 5$.

Every framed mapping class group of signature $\underline{k}$ is generated by an* explicit finite set of Dehn twists.

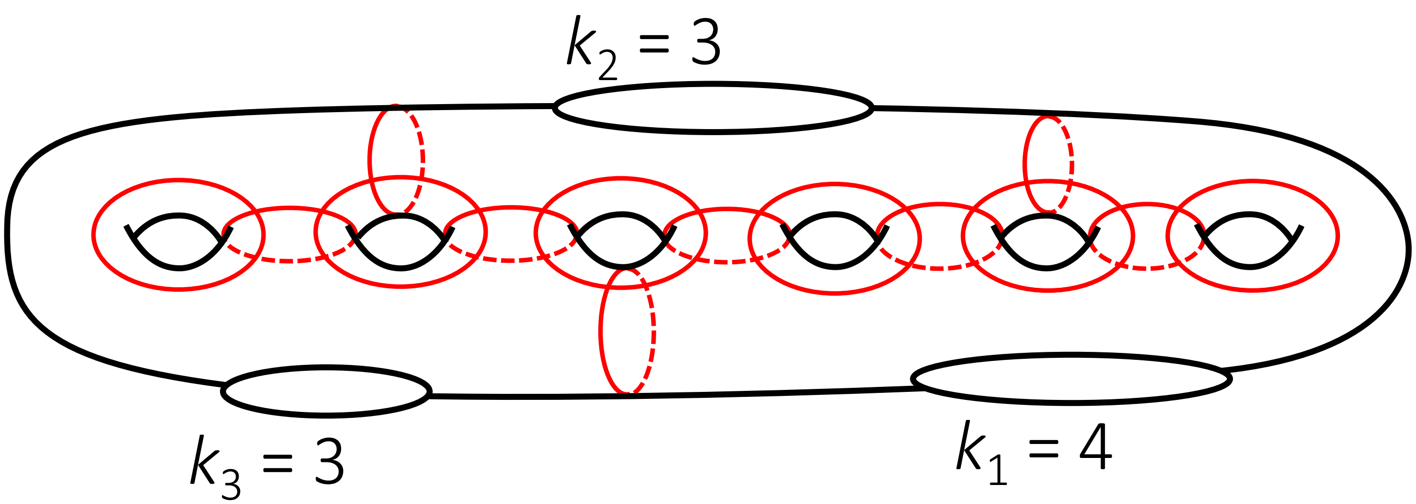
FMod: admissible twists
fMod: admissible + separating twists

*actually, we give a general inductive criterion for when a set of Dehn twists generates in terms of "stabilization."

Corollary: general criterion for twists to generate *closed* MCGs

Generating sets

<u>*e.g.*</u> signature (4,3,3) ⇝ genus 6



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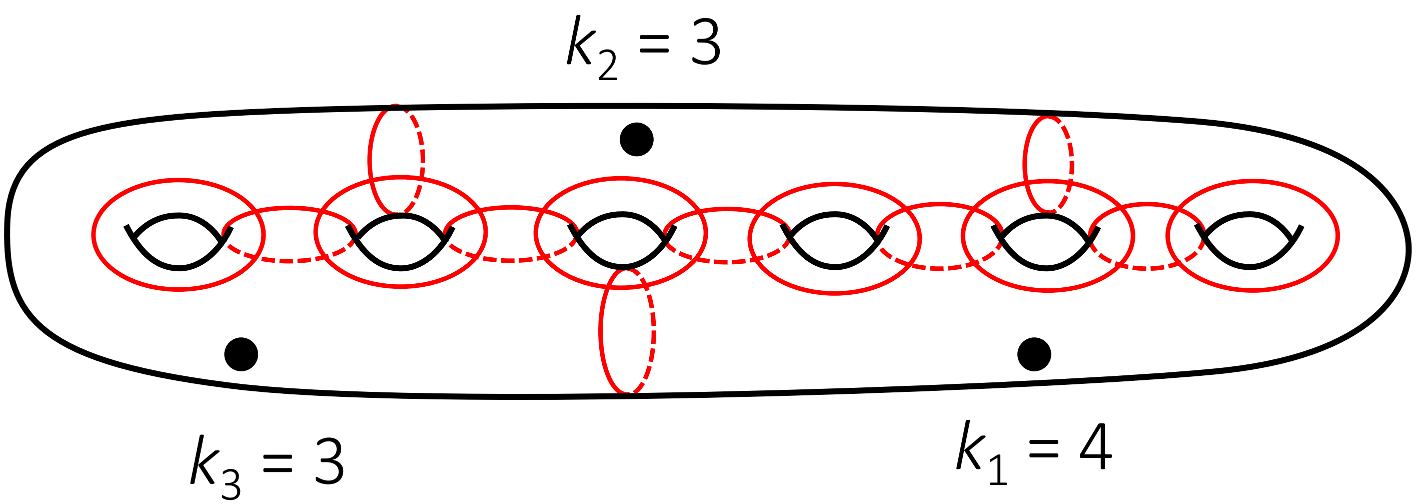

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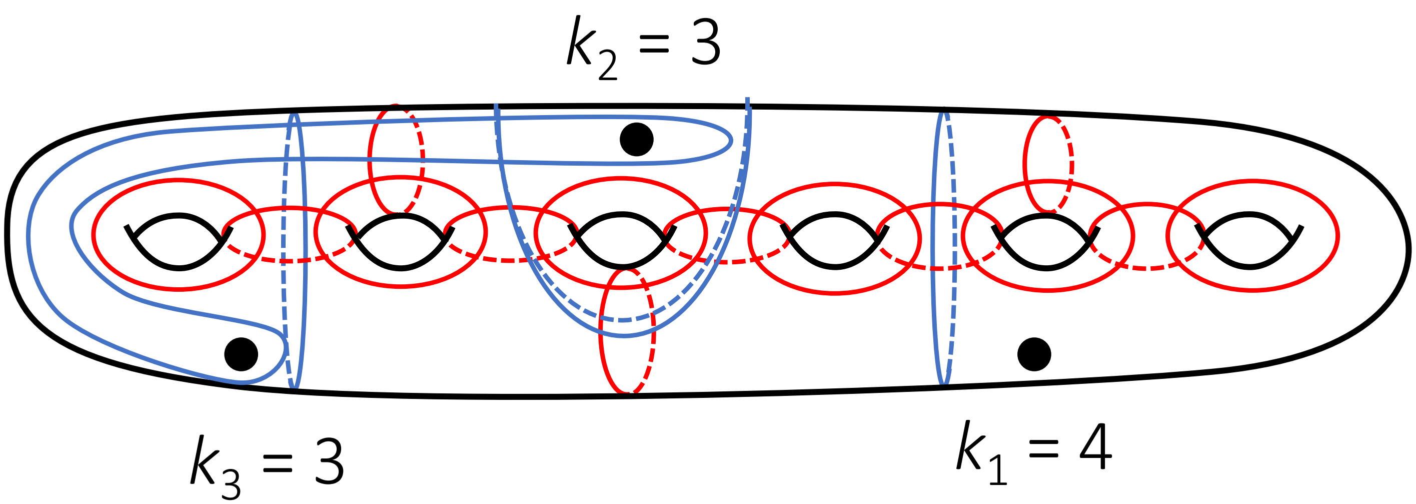

FMod: admissible twists

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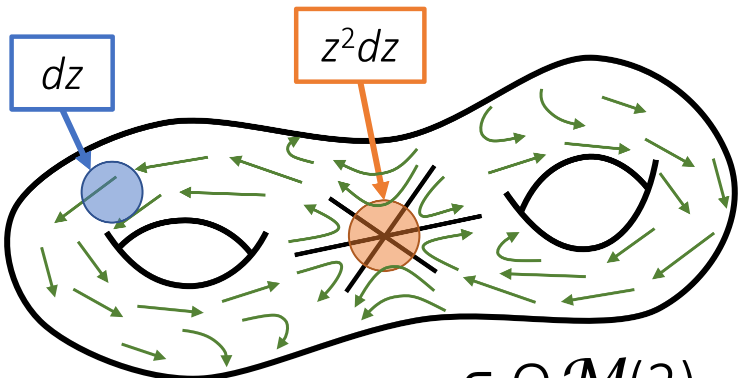
Holomorphic 1-forms



$\in \Omega\mathcal{M}(2)$

Locally, $\omega = dz$ or $z^n dz$

Vector field $1/\omega$ ⇜⇝ framing f of $S \setminus \text{Zeros}(\omega)$

$\Omega\mathcal{M}(k_1, \ldots, k_n)$ = "stratum"

= moduli space of ω's with zeros of orders $k_1, \ldots, k_n$

⇝ framing f has signature $(k_1, \ldots, k_n)$

Loops in $\Omega\mathcal{M}(k_1, \ldots, k_n)$ induce homeos of S preserving Zeros(ω)

$$\pi_1(\mathcal{H}, \omega) \longrightarrow \text{Mod}(S \setminus \text{Zeros}(\omega))$$

$\mathcal{H}$ = component of $\Omega\mathcal{M}(k_1, \ldots, k_n)$
(classified by Kontsevich–Zorich)

Theorem: [C.–Salter, ‘20]

Let $\mathcal{H}$ be a component of $\Omega\mathcal{M}(k_1, \ldots, k_n)$.*
Then the image of the monodromy

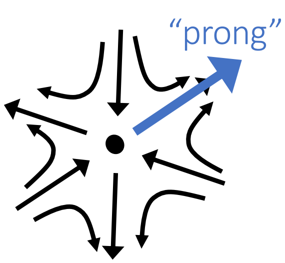
$$\pi_1(\mathcal{H},\omega) \longrightarrow \mathrm{Mod}(S \setminus \mathrm{Zeros}(\omega))$$

is the framed mapping class group preserving $1/\omega$.

Open Q: kernel? (we only know not injective in 2 very special cases!)

*for $g \geq 5$ and $\mathcal{H}$ non-hyperelliptic (the hyperelliptic case is both rare and classically understood)

Track:	Parallel transport:	Stabilizes:
zeros	surface with punctures	framing (~isotopy)
prong	surface with boundary	framing (~ relative isotopy)
all prongs	"pronged" surface	framing (~ pronged isotopy)
only surface [C.–Salter, '19]	closed surface	"*r*-spin structure" = framing mod $r = \gcd(\underline{k})$
only homology	$H_1(S, \mathrm{Zeros}(\omega))$	total mod 2 winding numbers



Tracking different data
⇝ different monodromy maps

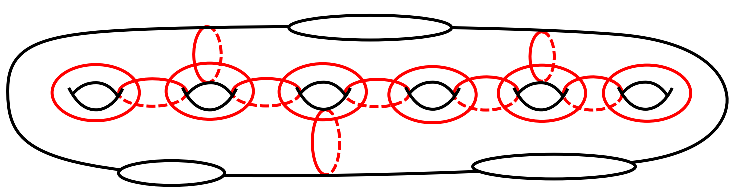
"Monodromy of a stratum always stabilizes some sort of framing"

Main theorems:

Let $(k_1, \ldots, k_n)$ be a partition of $2g-2$ with $g \geq 5$.

(Generating framed MCGs)

We give (many!) explicit generating sets for every framed mapping class group of signature $(k_1, \ldots, k_n)$.



(Characterization of monodromy)

The image of the map

$$\pi_1(\mathcal{H}, \omega) \longrightarrow \mathrm{Mod}(S \setminus \mathrm{Zeros}(\omega))$$

is the framed mapping class group preserving $1/\omega$.

Estimating Link Volumes via Subdivision

Lily Li

Tech Topology Conference, December 2020

Joint Work with Michele Capovilla-Searle, Darin Li, Jack McErlean, Alex Simons, Natalie Stewart, Miranda Wang
Mentor: Prof. Colin Adams

Lower Bounds on Volume

Theorem (Lackenby)

If L is a prime alternating link in S^3, then

$$v_3(t(L)-2)/2 \leq \mathrm{vol}(S^3 - K)$$

where $t(L)$ is the twisting number of L, and $v_3 \approx 1.0149$.

Theorem (Agol-Storm-Thurston)

Let M be a hyperbolic manifold, and let Σ be a totally geodesic surface in M. If M is cut along Σ and reglued to form a manifold M' that is also hyperbolic, then $\mathrm{vol}(M') \geq \mathrm{vol}(M)$.

Theorem

Let M be a Riemannian manifold and F the fix point set of an isometry of M. Then each connected component of F is a closed totally geodesic submanifold of M.

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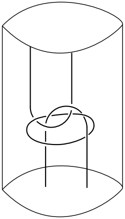
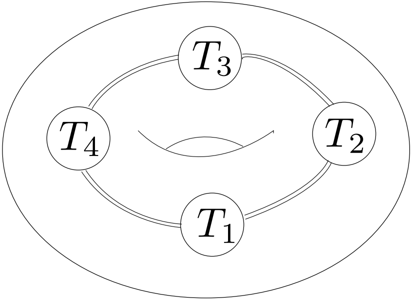


Definition

A cylindrical *tangle* is a disjoint embedding of finitely many circles and arcs ending at the "top/bottom caps."

Suppose link L in a solid torus decomposes into a cycle of tangles (T_i), with strands connecting adjacent tangles.



Links in solid tori

Definition

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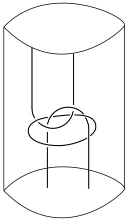

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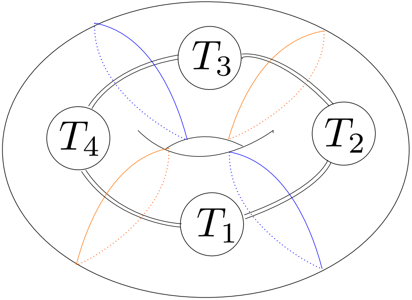


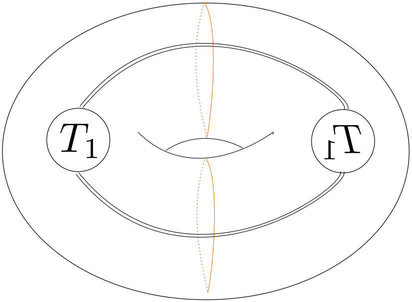
Hyperbolic tangles

Definition

A tangle T yields a link in a solid torus called the *double* $D(T)$.

T is *hyperbolic* if $D(T)$ is. In this case, we define the *volume*

$$\mathrm{vol}(T) := \frac{\mathrm{vol}(D(T))}{2}.$$



Theorem

Suppose L decomposes into a cycle $(T_i)_{i=1}^n$ of hyperbolic tangles. Then, L is hyperbolic with volume

$$\mathrm{vol}(L) \geq \sum_{i=1}^{n} \mathrm{vol}(T_i)$$

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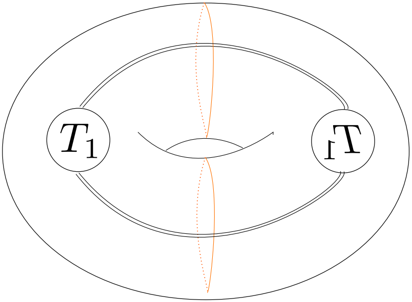

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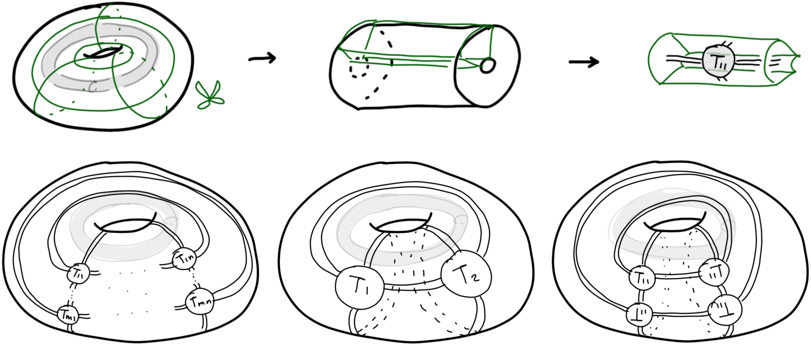
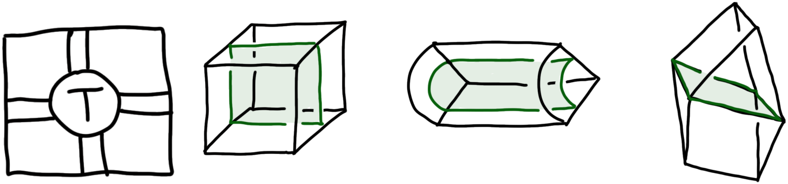
Square Tangles in a Thickened Torus



Square tangles

Definition

A square tangle $\mathcal{T}$ is the projection of a tangle living in a square, where the tangle $\mathcal{T}$ will have a collection of strands meeting each edge of the square.

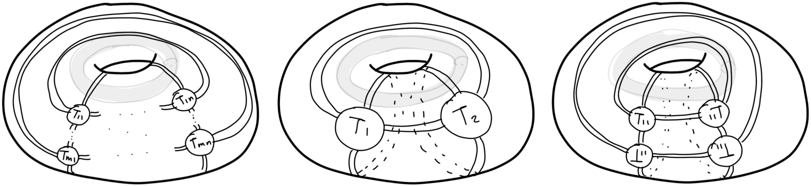


Square Tangles in a Thickened Torus

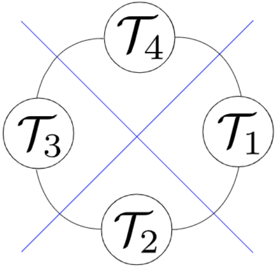
Theorem

Consider a link L in a thickened torus, that decomposes into an $n \times m$ grid of square tangles $\mathcal{T}_{i,j}$. Then:

$$vol(\mathcal{T}_{m \times n}) \geq \frac{1}{4} \sum_{i,j=1}^{n,m} vol_{C4}(\mathcal{T}_{i,j})$$



Necklace links in S^3

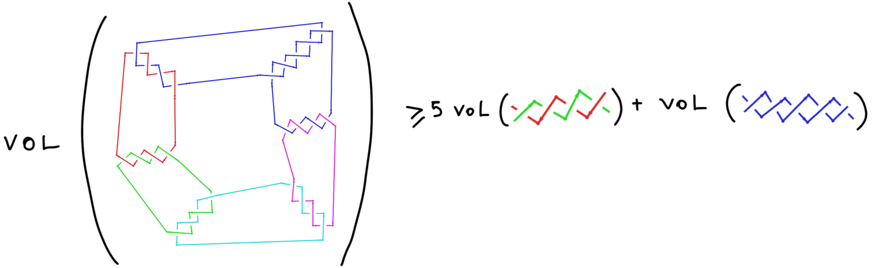


Theorem

Suppose L is a bracelet link made of a cycle $(\mathcal{T}_i)_{i=1}^m$ of $m \geq 2n$ saucer tangles such that each $\mathcal{T}_i$ is $2n$-hyperbolic. Then L is hyperbolic. If $m = 2n$, then the volumes satisfy

$$\mathrm{vol}(L) \geq \sum_i \mathrm{vol}_{2n}(\mathcal{T}_i).$$

An example



Lackenby's bound: 2.02988

Our bound: 32.7858

Actual volume: 32.9818

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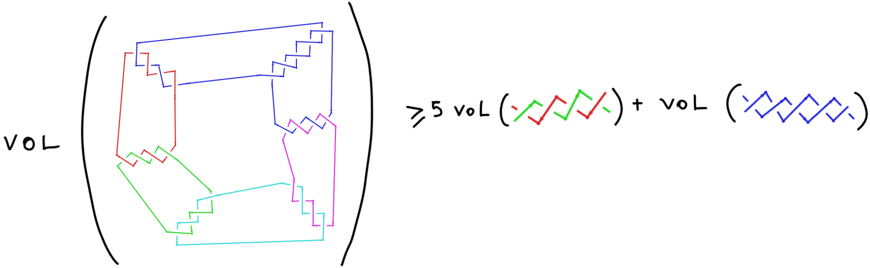

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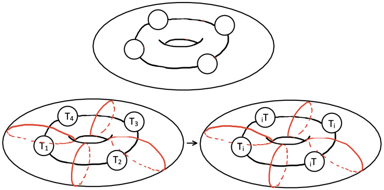
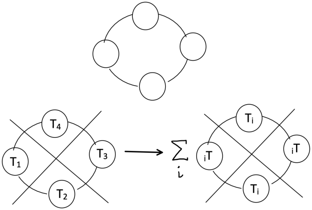
Other Linking Patterns

We've seen three configurations thus far. It turns out there are many more.





Other configurations:

Hexagonal tiling of the thickened torus;

Truncated square tiling of the thickened torus;

Archimedean Solids;

Pseudo-Anosov Stretch Factors and Coxeter Transformations

Joshua Pankau (Joint with Livio Liechti)

Tech Topology Conference

12/04/2020 - 12/06/2020

The University of Iowa
Visiting Assistant Professor

Preliminaries

Let f be a pseudo-Anosov element of $\mathrm{Mod}(S_g)$.

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Associated to f is a real number $\lambda > 1$ known as the **stretch factor** of f.

Theorem (Thurston 1974)
If $\lambda > 1$ is the stretch factor of a pseudo-Anosov map of S_g then λ is an **algebraic unit** where $[\mathbb{Q}(\lambda) : \mathbb{Q}] \leq 6g - 6$.

Fried's conjecture

Theorem (Fried 1985)
Every stretch factor is a **bi-Perron unit**.

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Open Question
Which bi-Perron units are stretch factors of pseudo-Anosov maps?

Fried's conjecture

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Every stretch factor is a **bi-Perron unit**.

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Open Question
Which bi-Perron units are stretch factors of pseudo-Anosov maps?

Fried's Conjecture
Every bi-Perron unit has a power that is a stretch factor.

Results

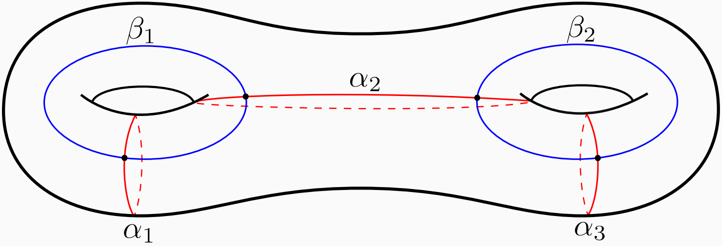
Theorem A (P. 2017)
Fried's conjecture is true for the class of Salem numbers.

Results

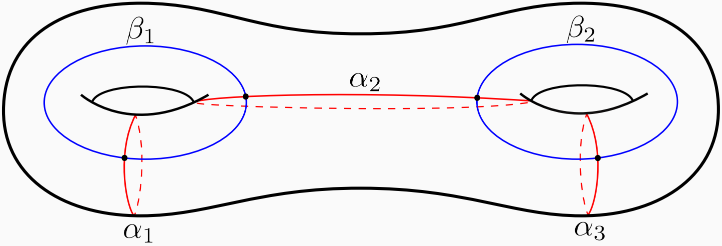
Theorem A (P. 2017)
Fried's conjecture is true for the class of Salem numbers.

Theorem B (Liechti, P. 2020)
Fried's conjecture holds for all bi-Perron units λ where $\lambda + \lambda^{-1}$ is totally real.

Example

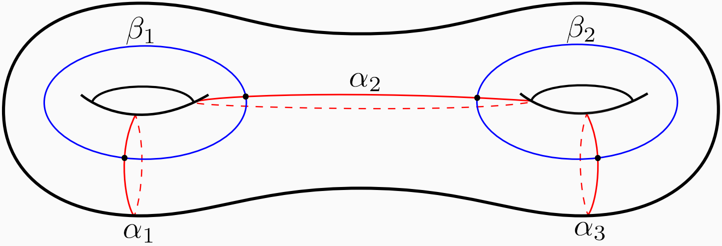


Example



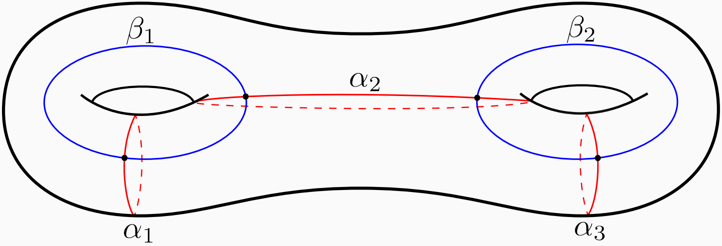
- Let $T_A = T_{\alpha_1}^2 \, T_{\alpha_2}^2 \, T_{\alpha_3}$ and $T_B = T_{\beta_1}^2 \, T_{\beta_2}^2$.

Example



- Let $T_A = T_{\alpha_1}^2 T_{\alpha_2}^2 T_{\alpha_3}$ and $T_B = T_{\beta_1}^2 T_{\beta_2}^2$.
- Thurston's construction guarantees that $T_A T_B$ is pseudo-Anosov.

Example



- Let $T_A = T_{\alpha_1}^2 T_{\alpha_2}^2 T_{\alpha_3}$ and $T_B = T_{\beta_1}^2 T_{\beta_2}^2$.
- Thurston's construction guarantees that $T_A T_B$ is pseudo-Anosov.
- Stretch Factor $\lambda = \dfrac{5 + \sqrt{17} + \sqrt{38 + 10\sqrt{17}}}{2}$, a Salem number.

Further Results

Proposition C (Liechti, P. 2020)
Let λ be a bi-Perron number. Then $\mathbb{Q}(\lambda + \lambda^{-1}) = \mathbb{Q}(\lambda^k + \lambda^{-k})$ for all positive integers k.

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Let λ be a bi-Perron number. Then $\mathbb{Q}(\lambda + \lambda^{-1}) = \mathbb{Q}(\lambda^k + \lambda^{-k})$ for all positive integers k.

Theorem D (Liechti, P. 2020)
For a bi-Perron number λ, the following are equivalent.

(a) For some positive integer k, λ^k is the stretch factor of a pseudo-Anosov homeomorphism arising from Thurston's construction.

(b) For some positive integer k, λ^k is the spectral radius of a bipartite Coxeter transformation of a bipartite Coxeter diagram with simple edges.

The End

Thank you!

Weinstein handlebodies of complements of toric divisors in toric 4-manifolds

joint work in progress with:
Bahar Acu, Agnès Gadbled,
Aleksandra Marinkovic, Emmy Murphy,
Laura Starkston, and Angela Wu

Orsola Capovilla-Searle

Duke University

November 24, 2020

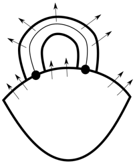
For any symplectic manifold (M^{2n}, ω) there exists a symplectic divisor, $(\Sigma^{2n-2}, i^*\omega) \subset (M^{2n}, \omega)$, such that the complement $M \backslash \nu(\Sigma)$ is an exact symplectic manifold and has a Weinstein handle decomposition [Donaldson, Giroux].

Goal: Find the Weinstein handlebody decomposition of $M \backslash \nu(\widetilde{\Sigma})$ for specific Σ and M.

Definition

A **Weinstein domain** $(X, \omega = d\lambda, \phi)$ is a compact exact symplectic manifold with boundary such that

1. There exists a Liouville vector field Z, defined by $\iota_Z \omega = \lambda$
2. Z is transverse to the boundary and therefore $\lambda|_{\partial X}$ is a contact form.
3. $\phi : X \to \mathbb{R}$ is a Morse function that is gradient like with respect to Z



Eliashberg gave a topological characterization of Weinstein $2n$-manifolds: you can only build them with handles of index $k \leq n$.
Weinstein handlebody diagrams for Weinstein 4-manifolds are given by projections of Legendrian links in $(\#^k(S^1 \times S^2), \xi_{std})$.

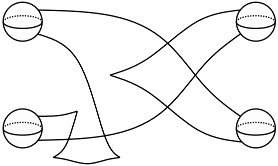


Figure: $D^* T^2$

A toric 4-manifold (M, ω) is a symplectic 4-manifold equipped with a effective Hamiltonian torus action. Then there exists a **moment map**

$$\Phi : M \to \mathbb{R}^2$$

that encodes the Hamiltonian torus action.

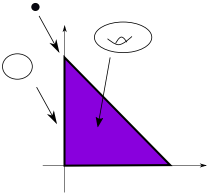


Figure: Moment map image of $\mathbb{C}P^2$

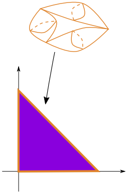


Figure: Toric divisor in $\mathbb{C}P^2$

Toric Divisors

The complement of any singular toric divisor $\Sigma \subset M$ is D^*T^2.
Goal: Consider smoothings $\widetilde{\Sigma}$ of Σ and if possible find the Weinstein handlebody decompostion of $M \backslash \nu(\widetilde{\Sigma})$

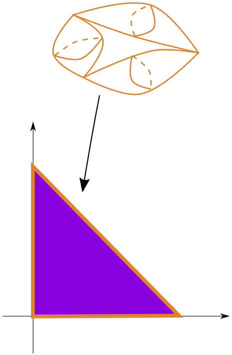


Figure: Singular toric divisor in $\mathbb{C}P^2$

The divisor $\widetilde{\Sigma}$ smoothed at the blue node has a complement given by attaching a two handle $h_{\Lambda_{(1,-1)}}$ to D^*T^2.

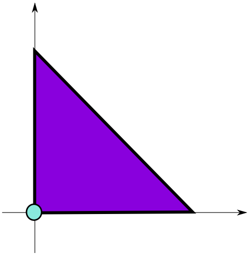


Figure: Difference of inward normals is $(1,-1)$

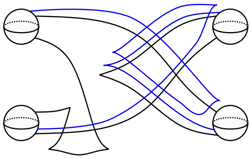


Figure: $D^*T^2 \cup h_{\Lambda_{(1,-1)}}$

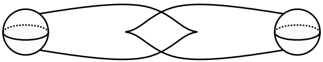


Figure: The complement of any toric divisor smoothed at one node.

Weinstein Complements of smoothed toric divisors

Theorem (Acu, C-S, Gadbled, Marinkovic, Murphy, Starkston, & Wu)

For certain toric 4-manifold X, the complement of the toric divisor smoothed at $(V_1, \ldots, V_n)$ nodes supports a Weinstein structure given by taking the completion of

$$D^* T^2 \cup h_{\Lambda_{(q_i, p_i)}}$$

where $h_{\Lambda_{(q_i, p_i)}}$ are 2-handles attached along the Legendrian conormal lift of $(q_i, p_i) \subset T^2$, and (q_i, p_i) are the difference of the inward normals at V_i

Thank you!

Integral Klein bottle surgeries and Heegaard Floer homology

Robert DeYeso III

Monday 23rd November, 2020

NC STATE UNIVERSITY

Why Dehn surgery?

For $K \subset S^3$, excise νK to obtain $S^3 \setminus \nu K$ and glue $D^2 \times S^1$ back in. Determined by $\mathrm{im}(S^1 \times \{\mathrm{pt}\}) = p\mu + q\lambda$; result is $S^3_{p/q}(K)$.

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- Open problems:
  - Cabling conjecture - Only cabled knots admit a reducible surgery.
  - Berge conjecture - Only Berge knots admit lens space surgeries.
  - Cosmetic Surgery conjecture - Different slopes never produce the same manifold.

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- Open problems:
  - Cabling conjecture - Only cabled knots admit a reducible surgery.
  - Berge conjecture - Only Berge knots admit lens space surgeries.
  - Cosmetic Surgery conjecture - Different slopes never produce the same manifold.
- If $S^3_{p/q}(K)$ contains a Klein bottle, then
  - p is divisible by 4.
  - If K is non-cabled, then $q = \pm 1$. (Teragaito)
  - $|p/q| \leq 4g(K) + 4$. (Ichihara & Teragaito)

Pairings

Let $X = S^3_8(K)$ with $g(K) = 2$ contain a Klein bottle. We have $X = (Y \setminus \nu J) \cup_h N$, where N is the twisted I-bundle over the Klein bottle.

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Theorem (D.)

If $X = (S^3 \setminus \nu J) \cup_h N$, then X is an L-space. Further,

- *If $J = U$, then $X = (-1; \frac{1}{2}, \frac{1}{2}, \frac{2}{5})$ and $K = T(2,5)$.*
- *If $J \neq U$, then J is a trefoil and K has the same knot Floer homology as that of $T(2,5)$.*

Heegaard Floer homology

To a 3-manifold Y, Ozsváth & Szabó associate a finitely-generated vector space over $\mathbb{F} = \mathbb{F}_2$ that decomposes as

$$\widehat{HF}(Y) = \bigoplus_{\mathfrak{s} \in \mathrm{Spin}^c(Y)} \widehat{HF}(Y, \mathfrak{s}).$$

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- Strong connection between $\widehat{HF}(S^3_{p/q}(K))$ and $\widehat{HFK}(K)$.

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Proposition

If $X = S^3_8(K)$ with $g(K) = 2$, then dim $\widehat{HF}(X, \mathfrak{s}) = 1$ for 5 of 8 spin^c structures $\mathfrak{s}$.

Bordered invariants as immersed curves

To a 3-manifold M with torus boundary, Hanselman, Rasmussen, and Watson associate an invariant $\widehat{HF}(M)$ in $T_M = \partial M \setminus \{z\}$.

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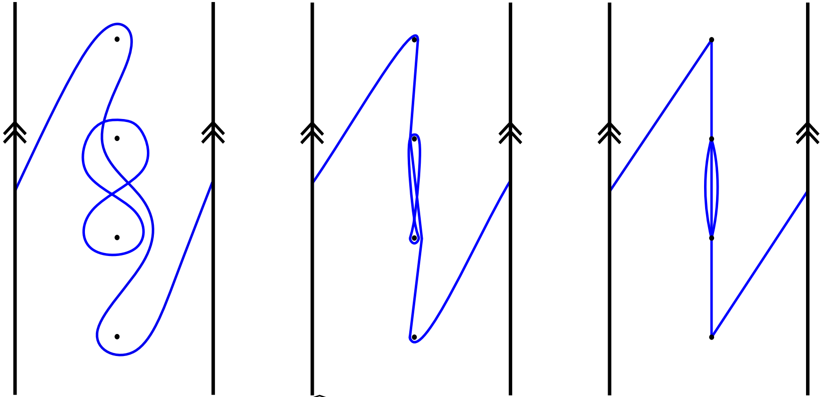

Figure: Pulling $\widehat{HF}(S^3 \setminus \nu(T(2,3)\#T(2,3)))$ tight

Pairing theorem

Theorem (Hanselman, Rasmussen, Watson)

Let $X = M_1 \cup_h M_2$. *Then*

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Pairing theorem

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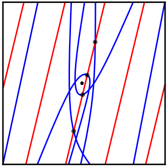
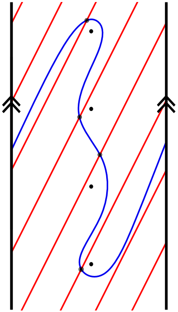
- Example for $S^3_4(T(2,5)) = (S^3 \setminus \nu T(2,5)) \cup_h (D^2 \times S^1)$.
- 4 lifts of $h(\widehat{HF}(D^2 \times S^1))$ needed to lift all intersections.
- $S^3_4(T(2,5))$ is an L-space.





Proof of main theorem

Let $X = S^3_8(K)$ with $g(K) = 2$ contain a Klein bottle, and be expressed as $X = (S^3 \setminus \nu J) \cup_h N$.

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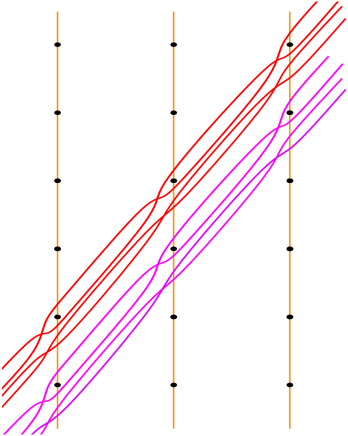

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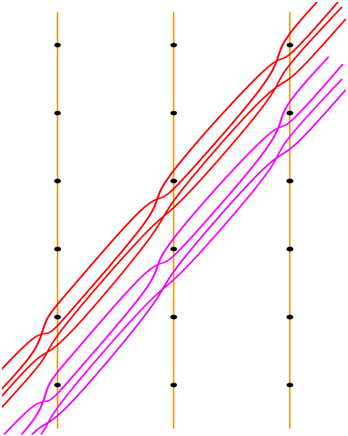

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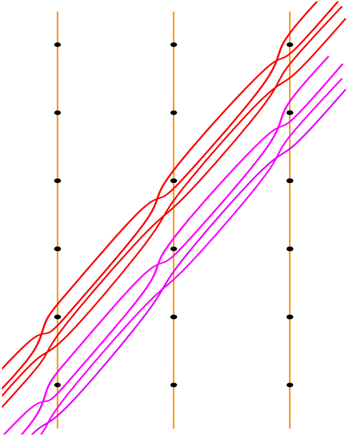

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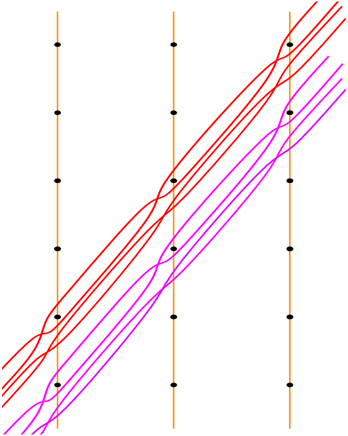


- $h(\widehat{HF}(N))$ fills with slope 2, and needs 2 copies to lift all intersections.
- Cannot have 4 of 8 curves intersecting $\widehat{HF}(S^3 \setminus \nu J)$ multiple times.
- $\widehat{HF}(S^3 \setminus \nu J)$ is heavily constrained. No interesting components and $g(J)$ must be small. □

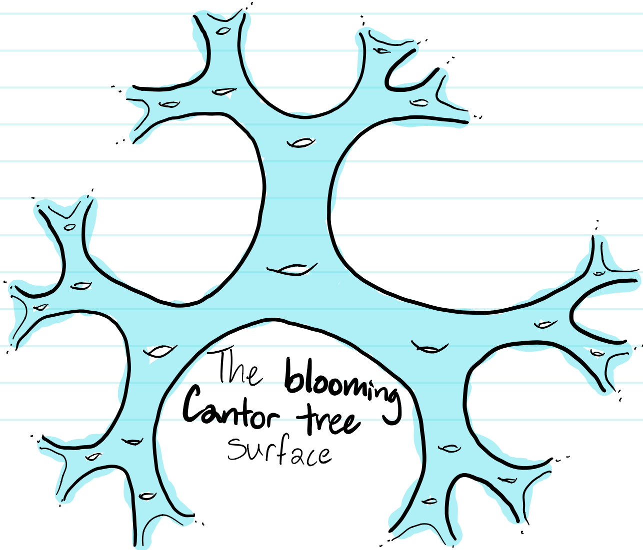
Nielsen Realization for Infinite-Type Surfaces

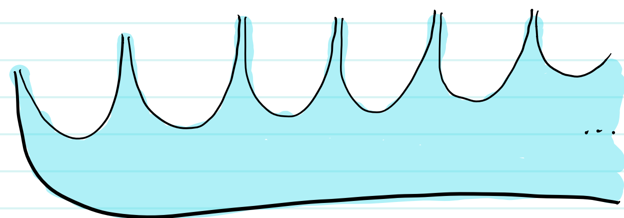
Rylee Lyman
Rutgers University–Newark

joint work with Santana Afton, Danny Calegari and Lvzhou Chen

An orientable surface is of **infinite type** if it has infinite genus or infinitely many punctures

Thm (Kerékjártó, Richards '63)
An orientable surface (without boundary) is classified by its **genus**, its **space of ends** (a closed subset of the Cantor set) and the closed subspace of **ends accumulated by genus**





The **flute** surface

Thm (Afton-Calegari-Chen-L '20) Let S be an orientable surface of infinite type. Finite subgroups of the mapping class group of S arise as groups of isometries of hyperbolic metrics on S.

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This theorem extends Kerckhoff's 1983 solution to the **Nielsen realization problem** to the infinite-type case.

The idea of the proof is to find an invariant exhaustion of S by finite-type subsurfaces and carefully apply Kerckhoff's theorem to the terms of the exhaustion.

Cor Let $P \subset S$ be an embedded pair of pants with boundary curves γ_1, γ_2 and γ_3.

$$\mathrm{stab}(\gamma_1) \cap \mathrm{stab}(\gamma_2) \cap \mathrm{stab}(\gamma_3)$$

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This corollary is key to proving the following.

Thm If G is a topological group containing torsion limiting to 1, then there is no continuous injection

$$G \hookrightarrow \text{Map}(S).$$

Thm Compact subgroups of $\text{Map}(S)$ are finite, and locally compact subgroups are discrete.