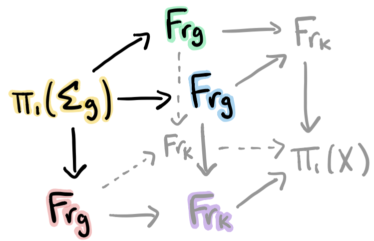

2-KNOT GROUP TRISECTIONS

Sarah Blackwell

University of Georgia

j.w. Kirby, Klug, Longo, Ruppik

2-KNOT GROUP TRISECTIONS

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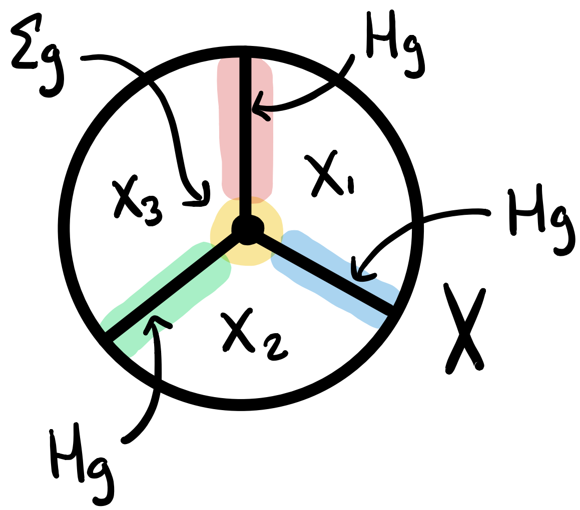
(g,k)-trisection of (closed) 4-mfld X = decomposition $X = X_1 \cup X_2 \cup X_3$ st
[Gay + Kirby]

1) $X_i \cong \natural^k (S^1 \times B^3)$
2) $X_i \cap X_j \cong H_g$
3) $X_1 \cap X_2 \cap X_3 \cong \Sigma_g$



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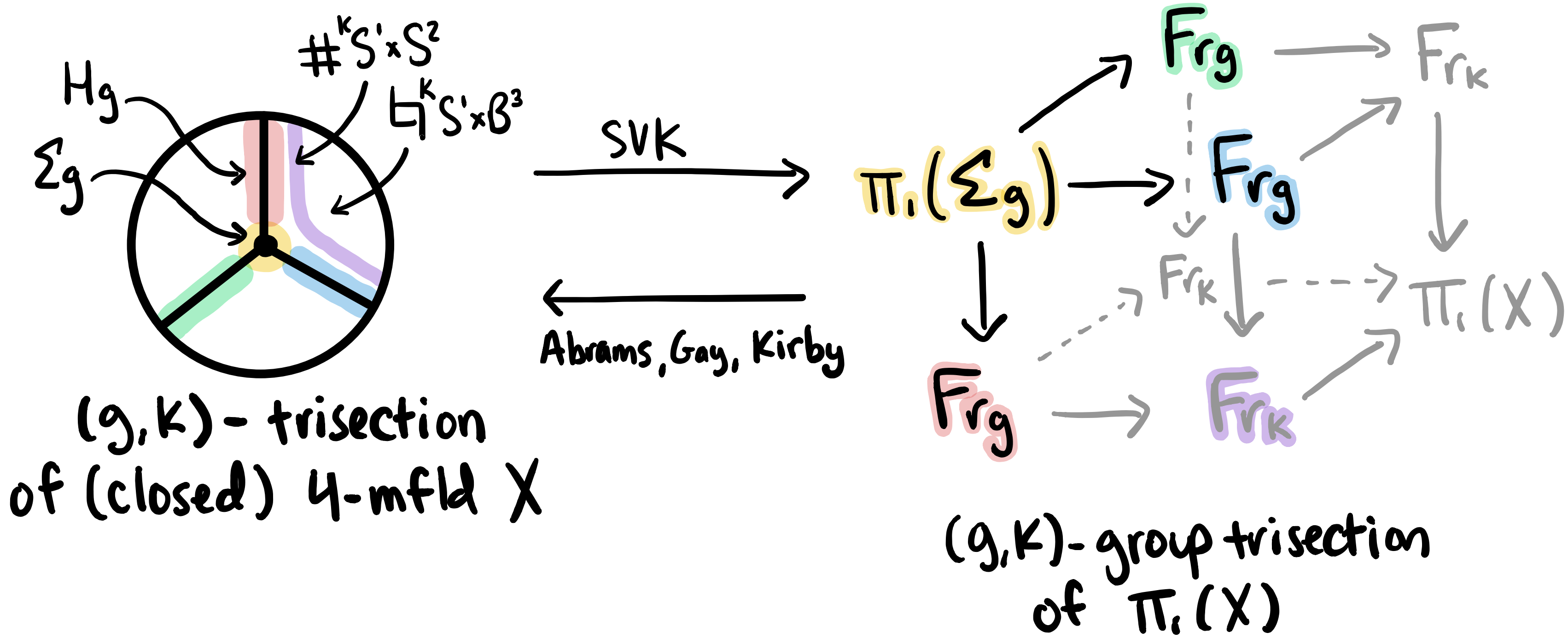

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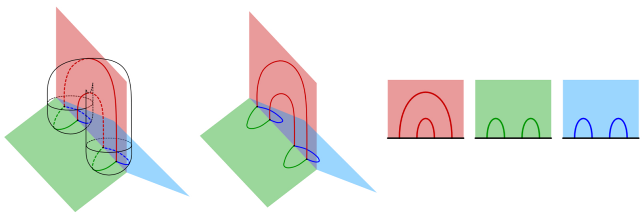
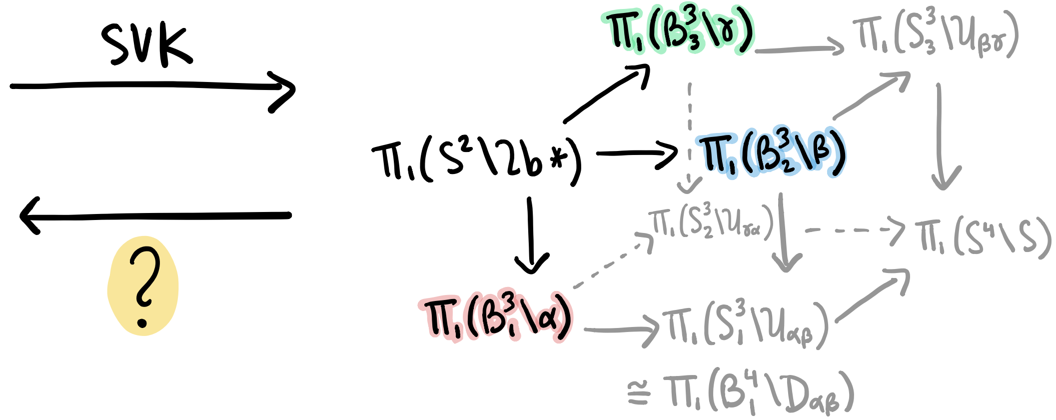


FIGURE 4. A 2–bridge trisection of an unknotted 2–sphere, depicted with the tri-plane in 3–space, along with the corresponding tri-plane diagram.

bridge trisection for a surface in S^4

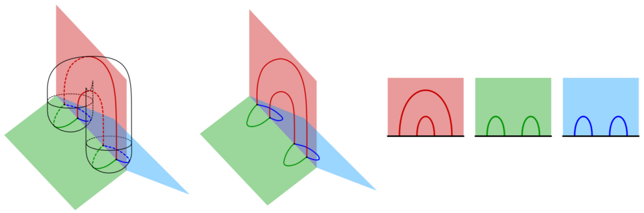
[Meier + Zupan]



Van Kampen diagram for the complement of a surface S in S^4

2-KNOT GROUP TRISECTIONS

j.w. Kirby, Klug, Longo, Ruppik



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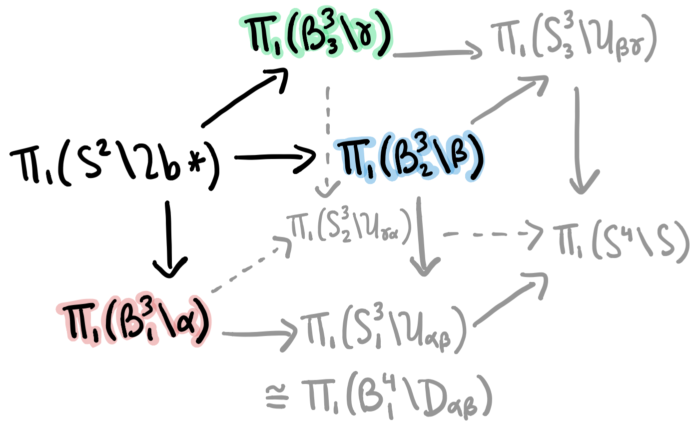
SVK →

← B.K.K.L.R.



Van Kampen diagram for the complement of a surface S in S^4

2-KNOT GROUP TRISECTIONS

Small Quotients of Braid Groups

Noah Caplinger
Joint with Kevin Kordek

Georgia Institute of Technology

December 2020

Main Question

Question

What is the smallest finite quotient of the braid group?

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Example 1. $B_n \to S_n$

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Example 1. $B_n \to S_n$

Example 2. $B_n \xrightarrow{\text{ab.}} \mathbb{Z} \to \mathbb{Z}/m\mathbb{Z}$.

Conjecture (Margalit)

For $n \geq 5$, S_n is the smallest non-cyclic quotient of B_n.

Main Theorem

Theorem

For $n = 5, 6$, S_n is the smallest non-cyclic quotient of B_n.

Totally Symmetric Sets

Definition (Kordek, Margalit)

Let G be a group. A subset $S = \{g_1, \ldots, g_k\} \subset G$ is said to be a totally symmetric set if

1. The elements of S pairwise commute.
2. Every permutation of S can be realized by conjugation in G.

Two Facts about Totally Symmetric Sets

Fact

If $f : G \to H$ is a homomorphism, and $S \subset G$ is totally symmetric, then $f(S)$ is totally symmetric of cardinality $|S|$ or 1.

Totally symmetric sets can "collapse" under homomorphisms.

Two Facts about Totally Symmetric Sets

Fact

If $f : G \to H$ is a homomorphism, and $S \subset G$ is totally symmetric, then $f(S)$ is totally symmetric of cardinality $|S|$ or 1.

Totally symmetric sets can "collapse" under homomorphisms.

Fact

A well-chosen totally symmetric set $X_n \subset B_n$ collapses under a quotient of B_n if and only if the quotient is cyclic.

Proof Strategy

If H has no totally symmetric sets of cardinality $|X_n|$, it cannot be a non-cyclic quotient of B_n.

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Better idea: Check for totally symmetric sets in simple groups of small order, then leverage this information to say something about braid groups.

Saying Something about Braid Groups

Theorem

For $n = 5, 6, 7, 8$, the alternating group A_n is the smallest non-trivial quotient of the commutator subgroup B_n'.

Saying Something about Braid Groups

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For $n = 5, 6, 7, 8$*, the alternating group* A_n *is the smallest non-trivial quotient of the commutator subgroup* B'_n*.*

Theorem

For $n = 5, 6$*,* S_n *is the smallest non-cyclic quotient of* B_n*.*

Link Detection Results for Knot Floer Homology

Fraser Binns,

joint work with Gage Martin

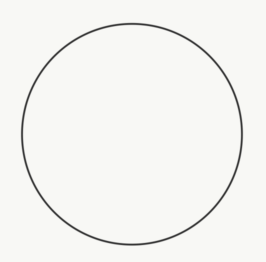
Boston College

Tech Topology Conference 2020

Can we distinguish links?

Question

If I meet two links in the wild, can I distinguish them?



What is knot Floer homology?

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Theorem (Ni, Ozsváth-Szabó)

$\widehat{\mathrm{HFK}}(L)$ *determines the genus of* L.

Theorem (Ghiggini, Ni)

$\widehat{\mathrm{HFK}}(L)$ *determines whether or not* L *is fibered.*

What is link Detection?

Definition

We say $\widehat{\mathrm{HFK}}$ *detects* L if whenever $\widehat{\mathrm{HFK}}(L') \cong \widehat{\mathrm{HFK}}(L)$, L' is isotopic to L.

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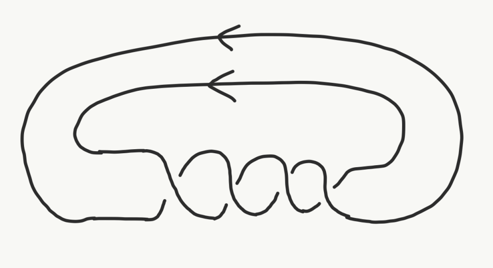
Theorem (B-Martin)

Knot Floer homology detects $T(2,4)$.



Detection Results for knot Floer homology

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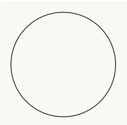
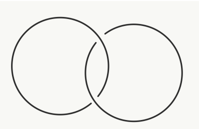


The unknot (Ozsváth-Szabó '04)



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Detection Results for knot Floer homology

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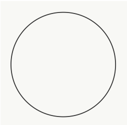

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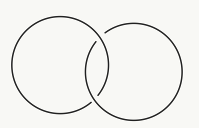

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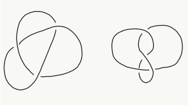
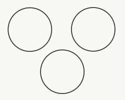


The trefoil, figure eight (Ghiggini '08)



Unlinks (Ni '14, Hedden-Watson '18)

Detection Results for knot Floer homology

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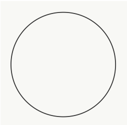

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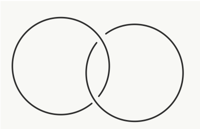

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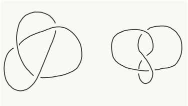

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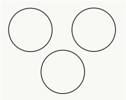

Unlinks (Ni '14, Hedden-Watson '18)

Knot Floer homology cannot distinguish:

- Infinitely many knots in each concordance class (Hedden-Watson '18)
- Non-trivial band sums of split links, where the bands differ by a twist (Wang '20)

Which links are good candidates for detection results?

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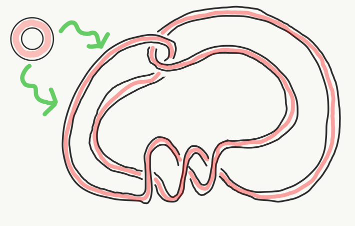
Definition

A 2-*cable link* is one which bounds an embedded annulus.



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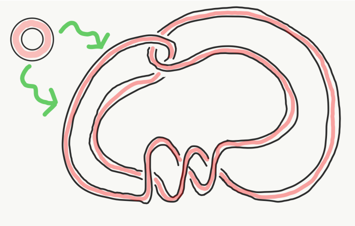
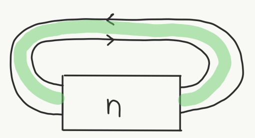


Remark

The torus links $T(2, 2n)$ are the 2-cables such that both components are unknotted.



Detection results

Theorem (B-Martin)

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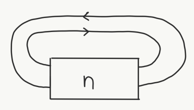
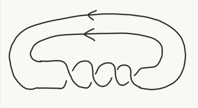


$T(2, 2n)$



$T(2, 4),\ T(2, 6)$

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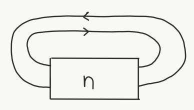

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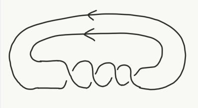

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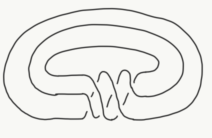
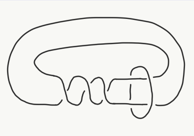


$T(3, 3)$



L7n1

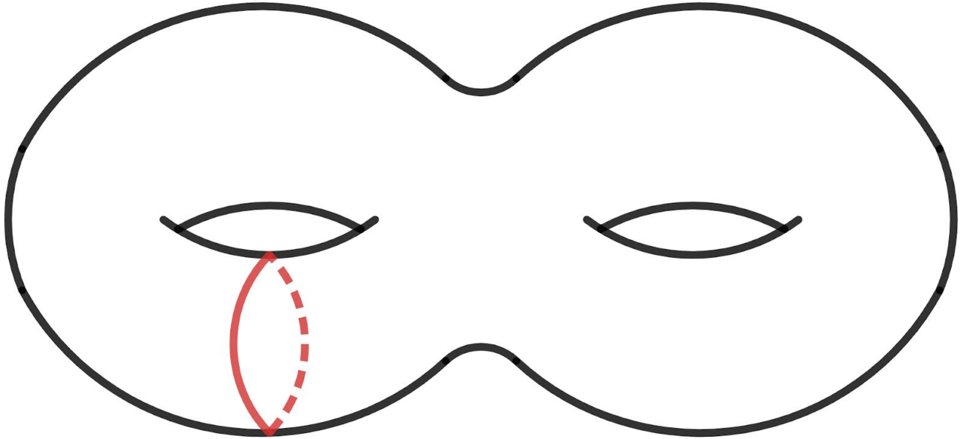
Mapping class groups vs. handlebody groups

Marissa Miller
University of Illinois at Urbana-Champaign

Mapping class group

S_g closed, orientable, genus g surface:

$$MCG(S_g) = \text{Homeo}^+(S_g)/\text{isotopy}$$



Curve graph $C(S_g)$

Vertices: Isotopy classes of essential simple closed curves

Edge: If two isotopy classes can be made disjoint

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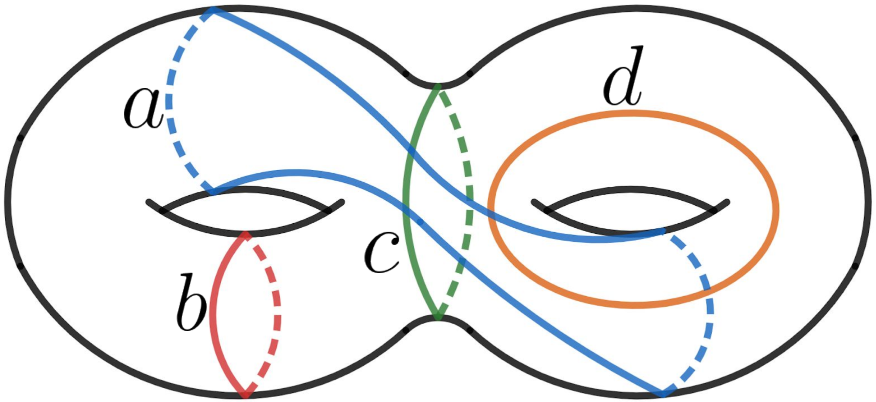
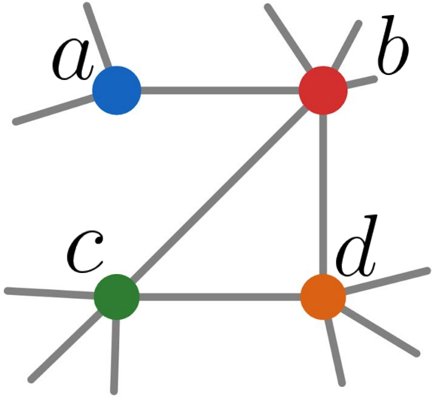
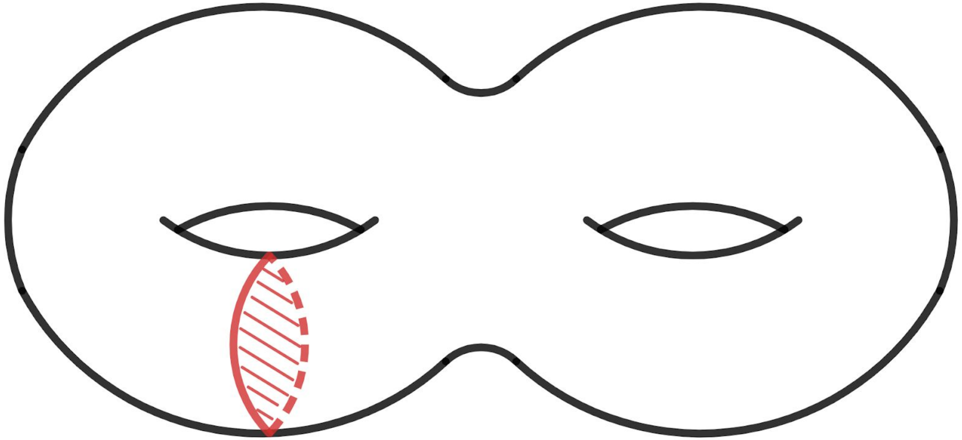




Handlebody group

Handlebody, V_g : 3-ball with g 1-handles attached (a 3-manifold)

$$H_g = MCG(V_g) = \text{Homeo}^+(V_g)/\text{isotopy}$$



Disk graph $D(V_g)$

Vertices: Isotopy classes of essential simple closed curves on ∂V_g bounding disks in V_g

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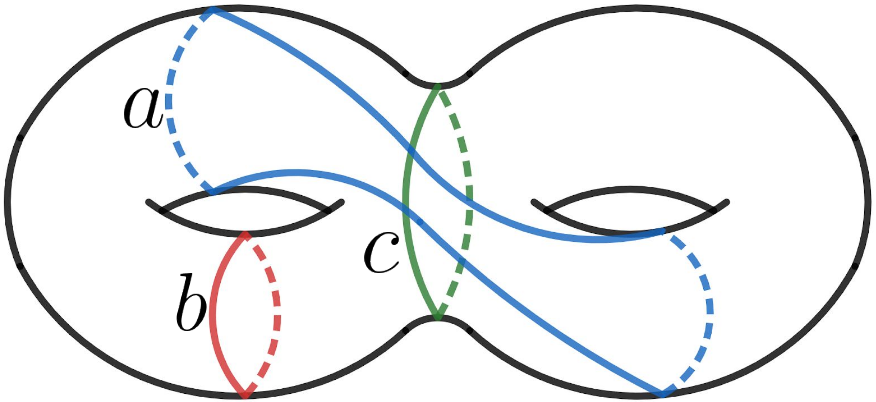

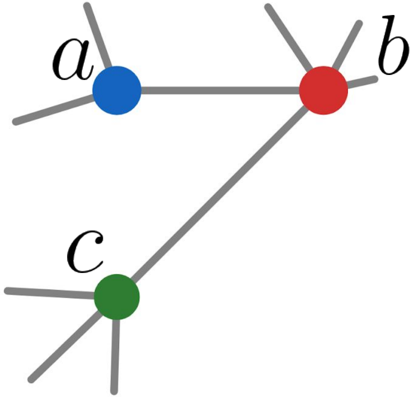

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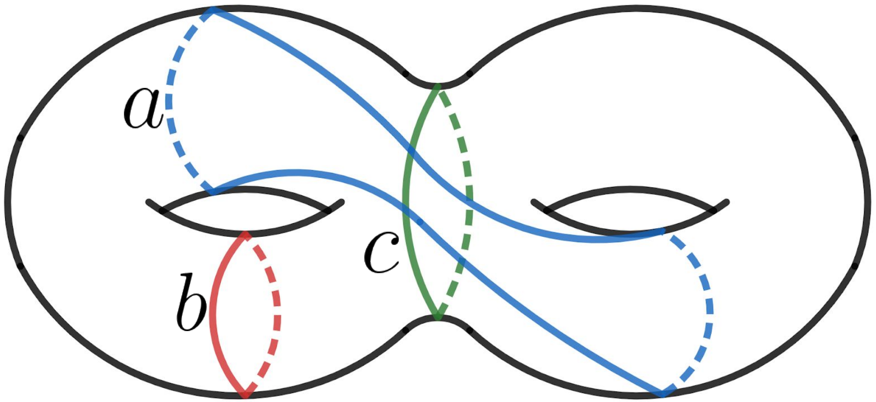

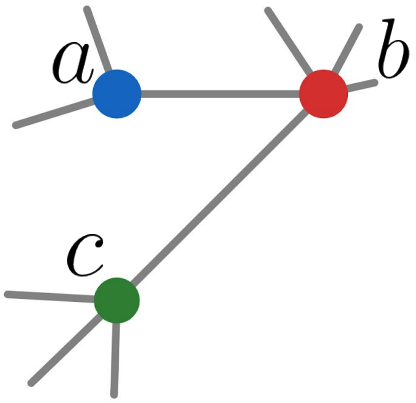

$D(V_g) \subset C(\partial V_g)$

A closer look...

1. $H_g < MCG(\partial V_g)$, but is badly distorted
2. $D(V_g) \subset C(\partial V_g)$, but is badly distorted

The geometries of H_g and $D(V_g)$ do not reflect the ambient geometries of $MCG(\partial V_g)$ and $C(\partial V_g)$

Hierarchically hyperbolic spaces?

HHS ≈ Almost hyperbolic; obstructed by product regions

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Mapping class groups: inspiration for HHSs (Behrstock-Hagen-Sisto)

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Mapping class groups: inspiration for HHSs (Behrstock-Hagen-Sisto)

Handlebody groups:

- Yes for genus two! (Miller)
- No for higher genus (Hamenstädt-Hensel, Behrstock-Hagen-Sisto)

Characterization of stable subgroups

Stable subgroup ≈ subgroups of finitely generated groups that exhibit hyperbolic-like behavior

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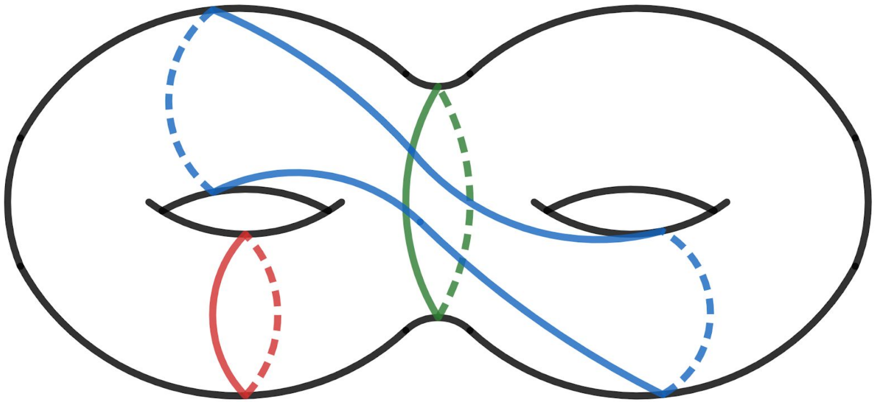
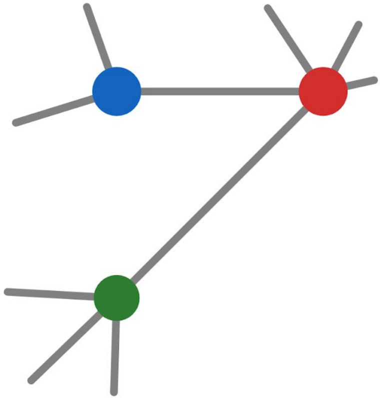
Mapping class groups: stable ⟺ quasi-isometrically embed in curve graph (Durham-Taylor, Hamenstädt, Kent-Leininger)

Handlebody groups:

- Genus two: stable ⟺ quasi-isometrically embed in disk graph (Miller)
- Higher genus: exist quasi-isometrically embedded subgroups that aren't stable (Miller)

Thank you!





Symmetric unions and reducible fillings

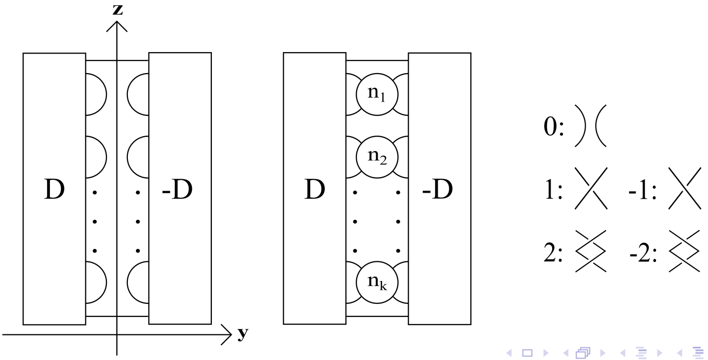
Feride Ceren Kose

Tech Topology Conference 2020

Symmetric unions

Definition

A *symmetric union* $(D \cup -D)(n_1, \ldots, n_k)$ $(n_i \in \mathbb{Z})$ is a knot diagram defined as follows:



Examples

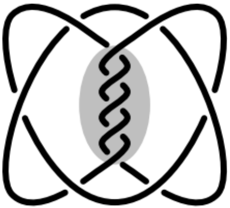


Figure: 11n139

Figure: 11n132

Symmetric unions

Theorem (Kinoshita-Terasaka '57, Lamm '00)

Symmetric unions are ribbon.

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- 122 of 137 prime ribbon knots with 11 and 12 crossings (Seeliger '14)
- all 2-bridge ribbon knots (Lamm '05)

Some classical results

Theorem (Fox-Milnor '58)

K is slice $\Rightarrow \Delta_K(t) = f(t)f(t^{-1})$

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Theorem (KT '57, Lamm '00)

$K = (D \cup -D)(n_1, \ldots, n_k)$ *with* $n_i \in 2\mathbb{Z} \Rightarrow \Delta_K(t) = (\Delta_D(t))^2$

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Theorem (KT '57, Lamm '00)

$K = (D \cup -D)(n_1, \ldots, n_k) \Rightarrow det(K) = (det(D))^2$

Minimal twisting number $tw(K)$

Definition

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Theorem (Tanaka '15)

$tw(11n132) = 2$

Main result

Theorem (Tanaka '19, K.'20)

Let K be a composite ribbon knot that admits a symmetric union diagram. If $tw(K) = 1$, then $K = K_1 \# K_2 \# - K_2$ where K_1 is a symmetric union with $tw(K_1) = 1$ and K_2 is a nontrivial knot.

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Corollary

$tw(3_1 \# 8_{10}) > 1$.

3-manifold topology

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Theorem (Gordon-Luecke '96)

Let M be an orientable and irreducible 3-manifold with a torus boundary. If $M(\pi)$ and $M(\gamma)$ are reducible for distinct slopes π and γ, then $\Delta(\pi, \gamma) = 1$.

Sketch of the proof

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- Two distinct reducible slopes $\frac{1}{n}$ and $-\frac{1}{n}$, but
  $\Delta(\frac{1}{n}, -\frac{1}{n}) = 2|n| \neq 1$

The end.

On embeddings of 3 manifolds in symplectic 4 manifolds

Anubhav Mukherjee

Georgia Institute of Technology

Dec.2020

Outline

Conjecture

Conjecture (Etnyre,Min,M.)

Every closed, oriented smooth 3-manifold smoothly embeds in a symplectic 4-manifold.

Why is such a Conjecture interesting?

The embedding of 3-manifolds in higher dimensional space has always been a fascinating problem.

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- Whitney's embedding theorem says that every closed oriented 3-manifold smoothly embeds in $\mathbb{R}^6$.

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- Hirsch improved this result by proving that every 3-manifold can be smoothly embedded in S^5.
- Meanwhile, Lickorish and Wallace proved that every 3-manifold can be smoothly embedded in some 4-manifold, and in fact, a generalization of their arguments shows that every 3-manifold can be smoothly embedded in the connected sum of copies of $S^2 \times S^2$.

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- Freedman proved that all integer homology 3-spheres can be embedded topologically, locally flatly in S^4.
- On the other hand, the Rokhlin invariant μ and Donaldson's diagonalization theorem show that some integer homology spheres cannot smoothly embed in S^4.

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Question

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Shiomi gave a negative answer to this question.
Thus one can ask, what is an interesting class of 4-manifolds in which all 3-manifolds embed?

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Main Results

Theorem (M.)

Given a closed, connected, oriented 3-manifold Y there exists a simply-connected symplectic closed 4-manifold X such that Y can be embedded topologically, locally flatly (i.e. it has collar neighbourhood) in X.

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Given a closed, connected, oriented 3-manifold Y there exists a simply-connected symplectic closed 4-manifold X such that Y can be embedded topologically, locally flatly (i.e. it has collar neighbourhood) in X. This embedding can be made a smooth embedding after one stabilization, that is Y can smoothly embed in $X\#(S^2 \times S^2)$.

Main Results

As an application of the proof of the last Theorem, we get followings...

Main Results

- Let Y_0 and Y_1 be smooth, oriented, closed 3-manifolds. A *cobordism* from Y_0 to Y_1 is a compact 4-dimensional smooth, oriented, compact manifold W with $\partial W = -Y_0 \sqcup Y_1$.

Main Results

- We say Y_0 and Y_1 are *R-homology cobordant*, if $H_*(W, Y_i; R) = 0$ for $i = 0, 1$.

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where R is a fixed commutative ring.

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where R is a fixed commutative ring.

- We give Θ^n_R the structure of a group where summation is given by the connected sum operation. The zero element of this group is given by the class of S^n, and the inverse of the class of $[Y]$ is given by the class of Y with reversed orientation.

Main Results

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- Livingston showed that these groups are generated by irreducible 3-manifolds.
- Myers showed that these groups are geneated by hyperbolic 3-manifolds.

Main Results

Theorem (M.)

The homology cobordism groups $\Theta^3_{\mathbb{Z}}$ and $\Theta^3_{\mathbb{Q}}$ are generated by Stein fillable 3-manifolds.

Main Results

Theorem (M.)

If an L-space Y smoothly embeds in a closed symplectic 4-manifold X then it has to be separating. Moreover, if $X = X_1 \cup_Y X_2$ then one of the X_i has to be a negative-definite 4-manifold.

Main Results

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There exists a 3-manifold Y which cannot be embedded(*) in any compact symplecteic 4-manifold with (weakly) convex boundary.

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Theorem (M.)

There exists a 3-manifold Y which cannot be embedded(*) in any compact symplecteic 4-manifold with (weakly) convex boundary.

- Example of 3-manifolds withouth symplectic fillings were known before by the work of Lisca–Matic, Etnyre–Honda.
- This above result is stronger in the sense that there exists 3-manifolds which cannot even embed(*) in (weak) filling of any 3-manifolds.

Main Results

The smooth v/s topoloical embeddings of 3-manifolds can be used to study exotic structure on 4-manifolds.

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Main Results

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There exists compact 4-manifolds with boundary X and X' such that $b_2(X) = b_2(X') = 1$ that are homeomorphic but not diffeomorphic.

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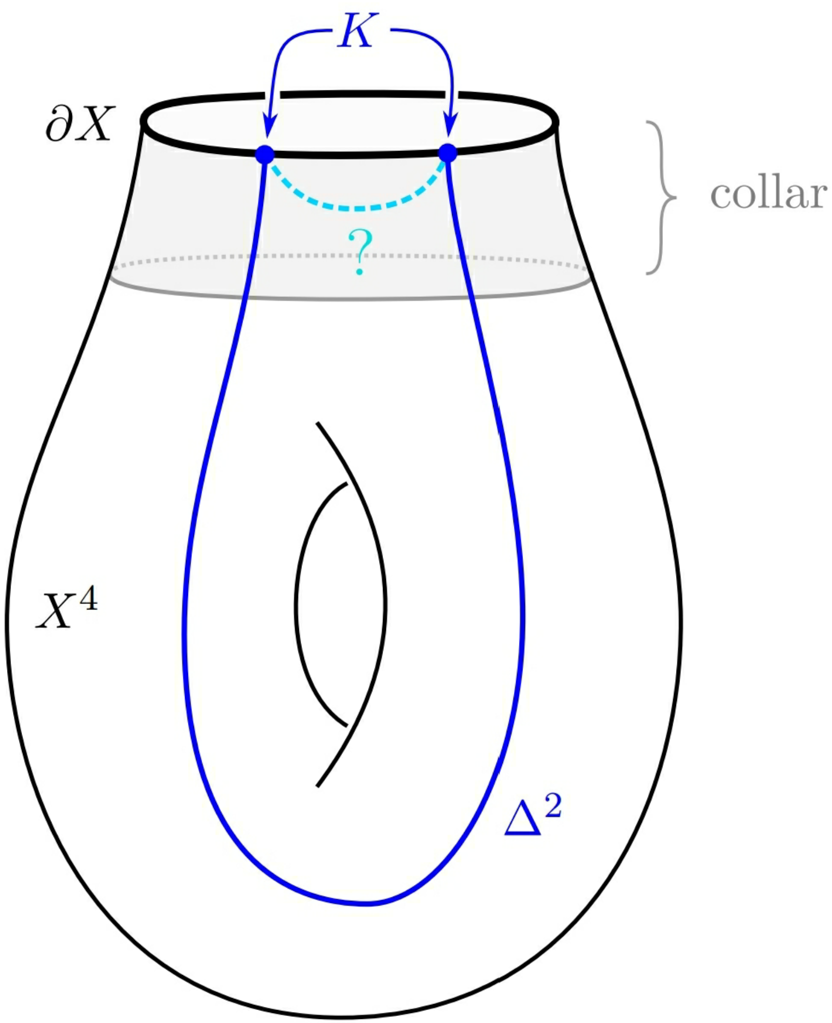
There exists compact 4-manifolds with boundary X and X' such that $b_2(X) = b_2(X') = 1$ that are homeomorphic but not diffeomorphic.

- Akbulut proved existence of such 4-manifolds first.
- The above is an alternative proof of that result.

Thank you!

Deep and shallow slice knots in 4-manifolds

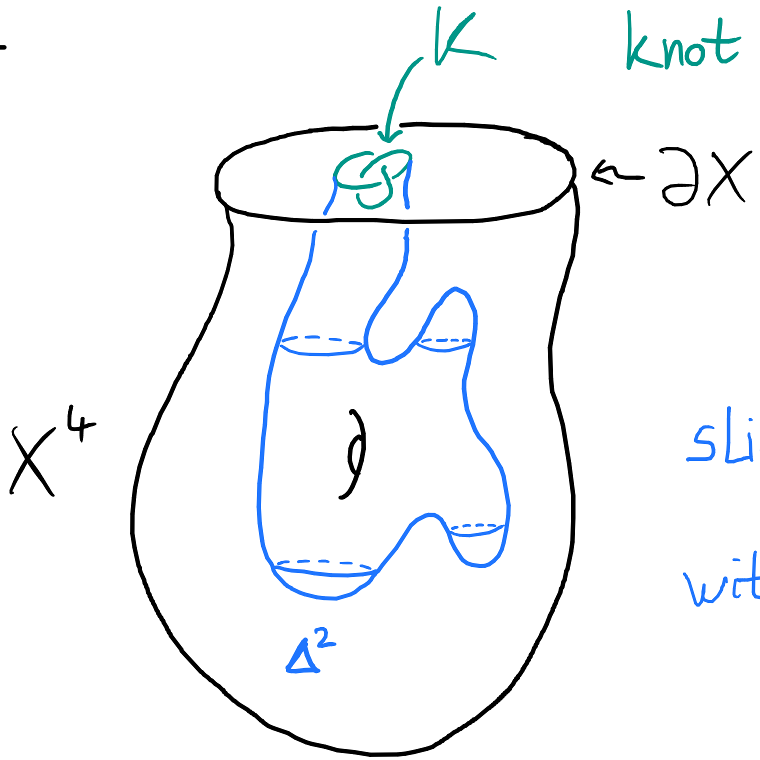


Joint work with Michael Klug

Benjamin Matthias Ruppik
PhD student @ Max-Planck-Institute for Mathematics, Bonn, Germany

Def.:

K knot in the boundary of a 4-mfld X^4

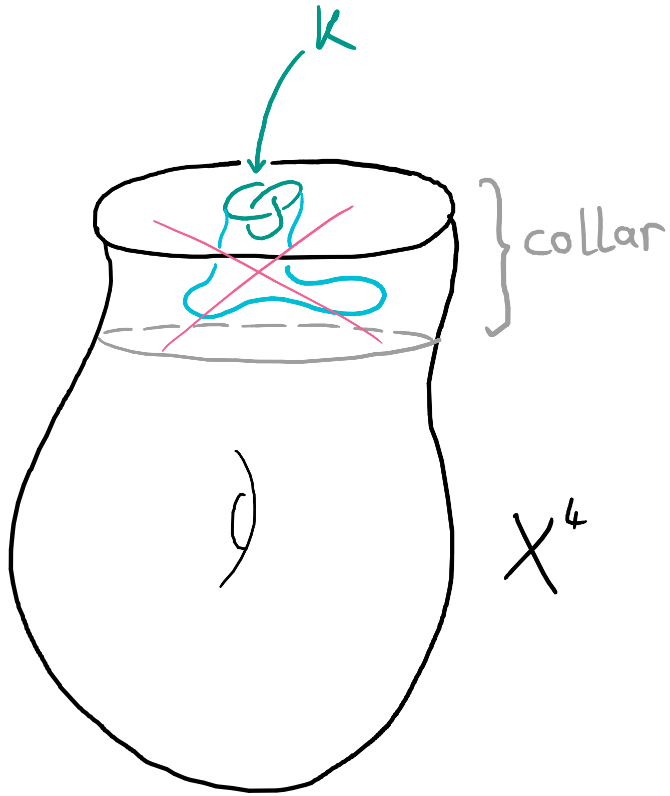


slice disk $\Delta^2 : \mathbb{D}^2 \hookrightarrow X$

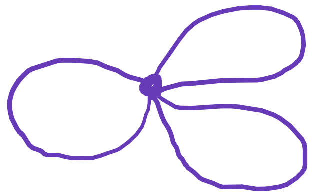
with $\partial \Delta^2 = K \subset \partial X$

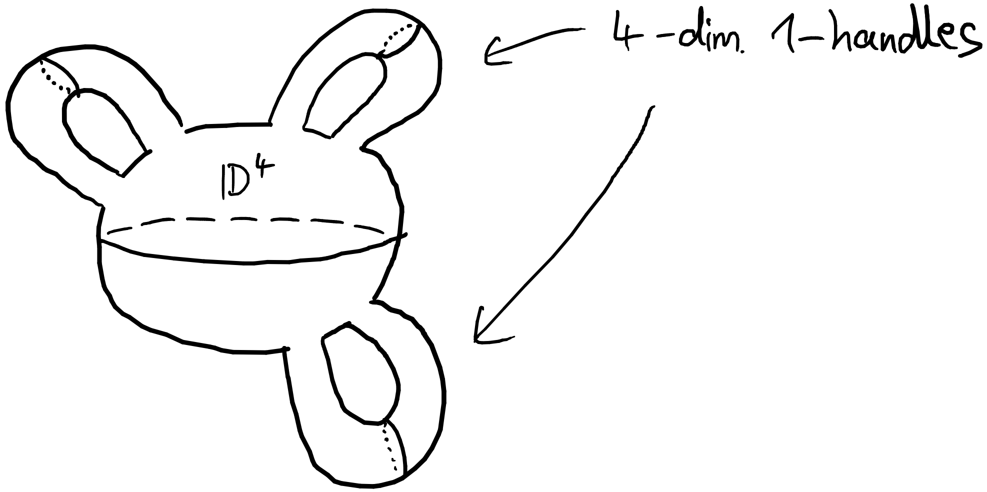
K is <u>deep slice in X</u> if the disk "needs to use the extra topology of X",

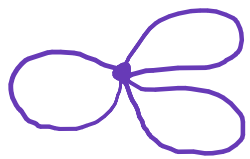
i.e. there is no slice disk for K in a collar $\partial X \times [0,1] \subset X$ of the ∂.



Non-example: There are no deep slice knots in $\natural^k S^1 \times D^3$.

$\natural^k S^1 \times D^3$ = thickening of 

= 

Any slice disk generically avoids the spine 

$\rightsquigarrow$ lives in a collar neighborhood of the boundary

Example: $X^4 = D^4 \cup (2\text{-handles})$ has deep slice knots in boundary (which are nullhomotopic in ∂X, but not contained in a 3-ball)

0-handle   at least one 2-h.

Two cases

$\pi_1(\partial X) = \{1\}$ and thus $\partial X \cong S^3$

We use a theorem of Rohlin on the genus of embedded surfaces representing 2-dim. homology classes in $\hat{X} = X \cup (4\text{-handle})$

$\pi_1(\partial X)$ non-trivial

Use Wall's self-intersection number with values in $\dfrac{\mathbb{Z}[\pi_1(\partial X)]}{\langle g = g^{-1}, 1 \rangle}$ of the track of a homotopy in $\partial X \times [0,1]$