Math 4318 **Practice Midterm Exam 1** **Spring 2011**

Name: ____________________________________

Signature: ________________________________

Instructions: Print your name and sign your signature to indicate that you accept the honor code. To receive full credit you must provide a proof that any answer you give is correct. You have 75 minutes to answer all the questions. *Good Luck*

Question	Max Point	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

1) Let $f : [a, b] \to \mathbb{R}$ and suppose that there is some M such that $|f'(x)| \leq M$. Prove using the definitions that f if Lipschitz and continuous on $[a, b]$.

2) Assuming that f' exists on $[a, b]$ and $\lim_{x \to c} f'(x) = L$ for some $c \in (a, b)$, prove that f' is continuous at c.

3) Let $f : [a, b] \to \mathbb{R}$ be an integrable function with $f(x) \geq 0$ for all $x \in [a, b]$.
a) If f is continuous at some $c \in (a, b)$ and $f(c) > 0$ show that

$$\int_a^b f(x)\,dx > 0.$$

b) If the set $C = \{x \in [a, b] : f(x) = 0\}$ has measure zero show that

$$\int_a^b f(x)\,dx > 0.$$

Hint: Does $[a, b]$ have measure zero?

4) Let $f : [0, 1] \to \mathbb{R}$ be the function that is 0 for all irrational numbers and $f(x) = x$ for all rational numbers. Prove that f is not integrable.
Hint: Show that the upper and lower Darboux integrals cannot be the same. Specifically show that any upper sum is bounded below by $\frac{1}{2}$.

5) Answer the following questions **T**rue or **F**alse. Circle either **T** or **F** to indicate your answer. You do not need to justify your answer.

1. If $|f|$ is integrable on $[a, b]$ then f is integrable on $[a, b]$.

   T **F**

2. If f is not integrable on $[a, b]$ then there are partitions $\mathcal{P}$ and $\mathcal{Q}$ of $[a, b]$ such that $L(f, \mathcal{Q}) > U(f, \mathcal{P})$

   T **F**

3. If a function is differentiable on an open interval I then it is continuous on I.

   T **F**

4. Sets of measure zero must be countable.

   T **F**

5. If a function if differentiable on an open interval I then its derivative is continuous on I.

   T **F**

6. If a function has bounded derivative on an interval then it is uniformly continuous on the interval.

   T **F**

7. Every integrable function has an anti-derivative.

   T **F**

8. The set of integrable functions form a vector space.

   T **F**

9. The product of integrable functions is integrable.

   T **F**

10. The composition of integrable functions is integrable.

    T **F**