Math 4318 **Practice Midterm Exam 2** **Spring 2011**

Name: ______________________________

Signature: ____________________________

Instructions: Print your name and sign your signature to indicate that you accept the honor code. To receive full credit you must provide a proof that any answer you give is correct. You have 75 minutes to answer all the questions. *Good Luck*

Question	Max Point	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

1) Let $g : \mathbb{R} \to \mathbb{R}$ be a uniformly continuous function. Define

$$G : C^0([a,b]) \to C^0([a,b]) : f \mapsto g \circ f.$$

If $C^0([a,b])$ has the sup-norm show that G is uniformly continuous.

2) Show that if $f_n : [a, b] \to \mathbb{R}$ is a sequence of differentiable functions that converge uniformly to f and the sequence f_n' converges uniformly to g then $f' = g$.

3) Let $\mathcal{F}$ be a family of equicontinuous functions from an interval $[a, b]$ to $\mathbb{R}$ that are bounded in the sup-norm. Define

$$F(x) = \sup\{f(x) : f \in \mathcal{F}\}$$

for all $x \in [a, b]$. Show that F is continuous.

4) a) Let $L : \mathbb{R}^n \to \mathbb{R}^m$ be a linear map and $b \in \mathbb{R}^m$. Define $f(x) = Lx + b$. What is the derivative of f at $y \in \mathbb{R}^n$?

b) Using the definition prove that your answer from a) is the derivative of f.

c) Compute the derivative of $f(x, y, z) = (xy^2, x + y, xy, 5 + x)$.

5) Answer the following questions **T**rue or **F**alse. Circle either **T** or **F** to indicate your answer. You do not need to justify your answer.

1. Analytic functions are smooth.

   T **F**

2. A set in $C^0([a,b])$ is compact if and only if it is closed and bounded. (Here we are using the sup-norm on C^0.)

   T **F**

3. If the derivative of a function $f : \mathbb{R}^n \to \mathbb{R}^m$ exists then all the partial derivatives of the coordinate functions exist.

   T **F**

4. The derivative of a differentiable function $f : \mathbb{R}^n \to \mathbb{R}^m$ is given by the Jacobian matrix (in the standard basis of $\mathbb{R}^n$ and $\mathbb{R}^m$).

   T **F**

5. A sequence of differentiable functions $\{f_n\}$ on a compact interval always has a subsequence that converge uniformly to some function on the interval.

   T **F**

6. If a sequence of continuous functions $\{f_n\}$ converges uniformly to f on $[a,b]$ then the sequence is equicontinuous.

   T **F**

7. The derivative of a function $f : \mathbb{R}^2 \to \mathbb{R}^3$ can be represented by a 3×3 matrix.

   T **F**

8. The normed vector space $(C^n([a,b]), \|\cdot\|_{C^n})$ is a Banach space.

   T **F**

9. If $f : \mathbb{R}^n \to \mathbb{R}^m$ and $g : \mathbb{R}^m \to \mathbb{R}^k$ are differentiable functions then $g \circ f$ does not have to be differentiable.

   T **F**

10. If a sequence $\{f_n\}$ of functions on a compact set is bounded in the sup-norm and equicontinuous, then the sequence converges uniformly.

    T **F**