1. Let $\Sigma$ be a regular, compact, orientable surface in $\mathbb{R}^3$ which is not homeomorphic to a sphere. Prove that there are points on $\Sigma$ where the Gaussian curvature is positive, negative and zero.

2. Determine the Christoffel symbols of a surface represented in the form $z = h(x, y)$.

3. In the local coordinates $f(u, v) = (u, v, h(u, v))$ write down the geodesic equations. That is if $\alpha(t) = f(a(t), b(t))$ is a geodesic. What equations must $a$ and $b$ satisfy?

4. Let $p \in \Sigma$ and $S_r(p)$ be the geodesic circle with center $p$ and radius $r$. Let $L$ be the length of $S_r(p)$ and $A$ the area of the region bounded by $S_r(p)$. Prove that

$$4\pi A - L^2 = \pi^2 K(p)r^4 + R$$

where $R$ is a function of $r$ satisfying

$$\lim_{r \to 0} \frac{R}{r^4} = 0.$$
