Math 4441 - Fall 2016
Formulas for Midterm 2

1. If $\boldsymbol{\alpha} : [a, b] \to \mathbb{R}^n$ is a regular parameterization of a curve C then the length of C is $\int_a^b \|\boldsymbol{\alpha}'(t)\|\, dt$.

2. If $\boldsymbol{\alpha} : [a, b] \to \mathbb{R}^n$ is a regular parameterization of a curve then $f(t) = \int_a^t \|\boldsymbol{\alpha}'(x)\|\, dx$ has an inverse $g : [0, l] \to [a, b]$ so that $\boldsymbol{\beta}(s) = \boldsymbol{\alpha}(g(s))$ is a parameterization of the same curve with $\|\boldsymbol{\beta}'(s)\| = 1$ (that is $\boldsymbol{\beta}$ is an arc length parameterization).

3. If $\boldsymbol{\alpha}$ is an arc length parameterization of a curve C, then $\boldsymbol{T}(s) = \boldsymbol{\alpha}'(s)$ is a unit tangent vector and $\boldsymbol{T}'(s)$ is perpendicular to $\boldsymbol{T}(s)$. The curvature of C at $\boldsymbol{\alpha}(s)$ is
$$\kappa(s) = \|\boldsymbol{T}'(s)\|$$
and the normal vector is $\boldsymbol{N}(s) = \frac{\boldsymbol{T}'(s)}{\|\boldsymbol{T}'(s)\|}$.

4. If $\boldsymbol{\alpha}$ is a curve in $\mathbb{R}^3$ then the binormal vector is $\boldsymbol{B} = \boldsymbol{T} \times \boldsymbol{N}$ and the torsion is
$$\tau(s) = -\boldsymbol{B}'(s) \cdot \boldsymbol{N}(s).$$

5. If $\boldsymbol{\alpha}$ is a regular parameterization of a curve C (but not necessarily an arc length parameterization), then the curvature of C at $\boldsymbol{\alpha}(t)$ is
$$\kappa(t) = \left\| \left(\frac{\boldsymbol{\alpha}'(t)}{\|\boldsymbol{\alpha}'(t)\|} \right)' \frac{1}{\|\boldsymbol{\alpha}'(t)\|} \right\|.$$

6. For a surface Σ in $\mathbb{R}^3$ in local coordinates
$$\boldsymbol{f} : V \to \Sigma$$
where V is an open subset of $\mathbb{R}^2$ with coordinates (u, v) we have the tangent space of Σ spanned by $\{\boldsymbol{f}_u, \boldsymbol{f}_u\}$. the first fundamental form in the basis $\{\boldsymbol{f}_u, \boldsymbol{f}_u\}$ is given by the matrix with entries $g_{11} = \boldsymbol{f}_u \cdot \boldsymbol{f}_u$, $g_{12} = g_{21} = \boldsymbol{f}_u \cdot \boldsymbol{f}_v$ and $g_{22} = \boldsymbol{f}_v \cdot \boldsymbol{f}_v$. The second fundamental from if given by $\begin{bmatrix} A & B \\ B & C \end{bmatrix}$ where $A = S_{\boldsymbol{p}}(\boldsymbol{f}_u) \cdot \boldsymbol{f}_u$, $B = S_{\boldsymbol{p}}(\boldsymbol{f}_u) \cdot \boldsymbol{f}_v$, and $C = S_{\boldsymbol{p}}(\boldsymbol{f}_v) \cdot \boldsymbol{f}_v$, where $S_{\boldsymbol{p}}$ is the shape operator.

7. With notation above the Gauss and mean curvature is give by
$$K = \frac{AC - B^2}{g_{11}g_{22} - g_{12}^2} \quad \text{and} \quad H = \frac{1}{2}\frac{Ag_{22} - 2Bg_{12} + Cg_{11}}{g_{11}g_{22} - g_{12}^2}.$$