# Math 500 - Fall 2001
# Homework 4

1) Let $X = [0,1], A = \{0,1\} \subset X$ and $Y = [4,5]$. Define the map $f : A \to Y$ by $f(0) = 4$ and $f(1) = 5$. Show that $X \cup_f Y$ is homeomorphic to $S^1$.

2) Let $X = \mathbb{R}^2 \setminus \{(0,0)\}$. Show that the decomposition space of $X$ defined as

$$\mathcal{D} = \{S_r | r > 0\},$$

where $S_r = \{(x,y) | x^2 + y^2 = r^2\}$, is homeomorphic to $\mathbb{R}$.

3) Given a collection of topological spaces $\{X_\alpha\}_{\alpha \in J}$ show that the product topology on $\prod_{\alpha \in J} X_\alpha$ is the smallest topology for which each of the projection maps is continuous.

Let $\rho$ be a bounded metric on $X = \mathbb{R}^n$ (by bounded I mean there is some constant $C$ so that $\rho(x,y) < C$ for all $x, y \in X$). Given two sets $A$ and $B$ in $X$ define

$$\rho(x, A) = \inf\{\rho(x,y) | y \in A\},$$

$$d_A(B) = \sup\{\rho(x,A) | x \in B\}$$

and

$$d(A,B) = \max\{d_A(B), d_B(A)\}.$$

Note: $d(\{x\},\{y\}) = \rho(x,y)$.

4) Let $\mathcal{F}$ be the set of all nonempty closed and bounded sets in $X$. Show $d$ is a metric on $\mathcal{F}$. This metric is called the Hausdorff metric on $\mathcal{F}$.

5) Given $A$ and $B$ in $\mathcal{F}$ show that $d(A,B) < \epsilon$ if and only if $A \subset U_\rho(B,\epsilon)$ and $B \subset U_\rho(A,\epsilon)$ where $U_\rho(A,\epsilon) = \{x \in X | \rho(x,A) < \epsilon\}$.