Math 500 - Fall 2001
Homework 8

1) Show there is a Cantor set C in the plane so that the graph of any continuous function from $[0,1]$ to $[0,1]$ intersects C.

2) Let D be a disk and I be an interval in ∂D. If Σ is a surface and $f : I \to \partial\Sigma$ is an embedding, then show the surface

$$\Sigma \cup_f D$$

is homeomorphic to Σ. HINT: it might be good to try to show that the space obtained from two disks by gluing them along intervals in their boundary is homeomorphic to a disk.

3) Let S_1 be a surface of genus g, $g \geq 1$, and S_2 be a surface that is the connected sum of $n, n \geq 1$, projective planes. Let S_i^0 be S_i with two disjoint disks removed. Define Σ to be $S_1^0 \cup S_2^0$ with ∂S_1^0 glued to ∂S_2^0. What is the surface Σ?

4) Show that for any surface Σ and points p and q in Σ there is a homeomorphism $h : \Sigma \to \Sigma$ such that $h(p) = q$.