Math 6458 - Spring 2009
Homework 2

Work all the problems, but carefully write up and turn in Problems 3, 5, 6, 7

1. Recall we denote $\underbrace{TM \otimes \ldots \otimes TM}_{l \text{ times}} \otimes \underbrace{T^*M \otimes \ldots \otimes T^*M}_{k \text{ times}}$ by $T^k_l M$ and $\Gamma(T^k_l M)$ denotes sections of this bundle. Given a tensor $\tau \in \Gamma(T^k_l M)$ we get a map

$$L_\tau : \underbrace{\mathcal{X}(M) \times \ldots \times \mathcal{X}(M)}_{k \text{ times}} \to \underbrace{\mathcal{X}(M) \times \ldots \times \mathcal{X}(M)}_{l \text{ times}}$$

that is multilinear over $C^\infty(M)$ by setting

$$L_\tau(v_1, \ldots, v_k)(p) = \tau_p(v_1(p), \ldots, v_k(p))$$

for all points $p \in M$ and vector fields v_i. (Here of course we are plugging the i^{th} vector into the i^{th} covector slot.)

Show that any map

$$L : \underbrace{\mathcal{X}(M) \times \ldots \times \mathcal{X}(M)}_{k \text{ times}} \to \underbrace{\mathcal{X}(M) \times \ldots \times \mathcal{X}(M)}_{l \text{ times}}$$

that is multilinear over $C^\infty(M)$ is of the form L_τ for some $\tau \in \Gamma(T^k_l M)$. In other words show that sections of the tensor bundle $T^k_l M$ are in one-to-one correspondence with such multilinear maps.

2. Work problem 4-2 in Lee's book.

3. Work problem 4-4 in Lee's book.

4. Work problem 4-5 in Lee's book.

5. Work problem 5-2 in Lee's book.

6. Work problem 5-4 in Lee's book.

7. Work problem 5-6 in Lee's book.

8. Work problem 5-9 in Lee's book.