Math 4432 - Spring 2006
Practice Final Exam

These are practice problems for the final exam. Please make sure you can work all of these problems. If you have any questions on the talk to me before the exam.

A1) Let X be a topological space. Set $\Delta = \{(x, x) \in X \times X\}$. Show X is Hausdorff if and only if Δ is closed in $X \times X$.

A2) Show an open connected subset of a surface is path connected.

A3) Show that the letters O, X and Y, thought of as subspaces of $\mathbb{R}^2$, are not homoemorphic.
(Here I mean that O is a letter in $\mathbb{R}^2$ and we think of it as a subspace. That is O is simply a circle in $\mathbb{R}^2$.)

B1) Let Σ_1 and Σ_2 be two surfaces. Compute the Euler characteristic of $\Sigma_1 \# \Sigma_2$. (Justify your computation.)

B2) Remove a small neighborhood from the center circle in a Möbius band which is itself a smaller Möbius band. Show that the resulting surface is an annulus.

B3) Let γ be an embedded circle in a surface $\Sigma_{n,m}$. (Recall, $\Sigma_{n,m}$ is the connected sum of n tori with m disjoint disks removed.) Remove a small open neighborhood of γ from $\Sigma_{n,m}$. Identify the resulting surface, assuming that it is connected.

C1) Compute the fundamental group of the connected sum of two 3-manifolds M_1 and M_2 in terms of the fundamental group of M_1 and M_2. (Recall to form the connected sum remove an open embedded ball form both M_1 and M_2 then glue the resulting manifolds together along the S^2's in their boundaries created by removing the balls.)

C2) Show that any compact surface with boundary is homotopy equivalent to a bouquet of circles. Compute the fundamental group of $\Sigma_{n,m}$.

C3) Show that O and X, thought of as subspaces of $\mathbb{R}^2$, are not homotopy equivalent.