1. (13 points) Economists are studying the spending habits of a large group of people. Let \( x \) be the total disposable income of this entire group and let \( C \) be the total amount spent by the entire group. The relationship between \( x \) and \( C \) is: \( C(x) = 0.75x + 6 \), where both \( x \) and \( C \) are measured in billions of dollars. (a) (5) Calculate \( C(12) \) and \( C(50) \). (b) (4) In terms of the definitions of \( x \) and \( C \), interpret your answer to \( C(50) \). (c) (4) In terms of \( x \) and \( C \), explain why your answer to \( C(12) \) does NOT seem to be correct. (Use the correct units in all answers.)

Solution: (a) \( C(12) = 15 \) billion dollars & \( C(50) = 43.5 \) billion dollars

b. If the total disposable income of the entire group is 50 billion dollars, then the total amount spent by the group is 43.5 billion dollars

c. \( x = 12 \) ($) is the total disposable income; \( C(12) = 15 \) ($). It seems to be a contradiction that the group can spend more money that it has at its disposal.

(TA's: You are going to need to read (b & c) their answers carefully to see if they understand what they are saying; it does not need to be exactly as mine)

2. (13 points) A study by the Chamber of Commerce of a small city estimates that the population \( P \) of the city is related to \( x \), the number of months from now by the relationship: \( P(x) = 50000 + 30x^{3/2} + 20x \). a. (5) What is the population now? b. (8) By how much will the population increase in the next 9 months?

Solution: a. now means that \( x = 0 \) ==> population now = \( P(0) = 50,000 \) people

b. In 9 months the population will be \( P(9) = 50000 + 30 \cdot 9^{3/2} + 20 \cdot 9 = 50,990 \) people

==> increase in population = \( P(9) - P(0) = 50990 - 50000 = 990 \) people

3. (13 points) The monthly variable costs (V) to produce \( x \) units of a certain product is given by: \( V(x) = 0.000003x^3 - 0.03x^2 + 200x \) and the monthly fixed cost is $100,000. a. (7) Find a function that relates \( C = \) total cost to \( x \). b. (6) What is the total cost if the company produces 2000 units/month.

Solution: a. total costs = variable costs + fixed costs
\[ C(x) = 0.000003 \cdot x^3 - 0.03 \cdot x^2 + 200 \cdot x + 100000 \]

b. If \( x = 2000 \), then \( C(2000) = 0.000003 \cdot 2000^3 - 0.03 \cdot 2000^2 + 200 \cdot 2000 + 100000 \)

\[
= 404,000
\]

4. (15 points) Compute the following limits:

a. \[ \lim_{t \to 3} (4 \cdot t^2 - 2 \cdot t + 1) = (4 \cdot 3^2 - 2 \cdot 3 + 1) = 31 \]

b. \[ \lim_{x \to -2} \left( \frac{x^2 - 4}{x + 2} \right) = \lim_{x \to -2} \frac{(x-2)(x+2)}{(x+2)} = \lim_{x \to -2} (x-2) = -4 \]

c. \[ \lim_{x \to 1} \left( \frac{\sqrt{x} - 1}{x - 1} \right) \cdot \frac{\sqrt{x} + 1}{\sqrt{x} + 1} = \lim_{x \to 1} \frac{(x-1)}{(x-1)(\sqrt{x} + 1)} = \lim_{x \to 1} \left( \frac{1}{\sqrt{x} + 1} \right) = \frac{1}{2} \]

5. (13 points) The concentration (\( C \) – measured in \( \frac{mg}{cm^3} \)) of a drug in a patient's blood \( t \) hours after injection is given by: \[ C(t) = \frac{2 \cdot t^2}{100 \cdot t^2 + 100} \].

a. (9) Find the horizontal asymptote (\( t \to \infty \)) of the function \( C(t) \);

b. (4) Interpret your answer to a in terms of the concentration of the drug and the time (use the correct units in your answer).

Solution: a. To find the H.A., compute:

\[ \lim_{t \to \infty} \left( \frac{2 \cdot t^2}{100 \cdot t^2 + 100} \right) = \lim_{t \to \infty} \left( \frac{2 \cdot t^2}{100 \cdot t^2} \right) = \frac{2}{100} = 0.02 \frac{mg}{cm^3} \]

b. After a significant amount of hours (\( t \)) has passed, the concentration of the drug in the patient will be approximately \( 0.02 \frac{mg}{cm^3} \).

TA's – see the note on problem 1.

6. (13 points) The demand function for a certain product has the form:

\[ p = \sqrt{-a \cdot x^2 + b} \], where \( x \) is the demand (in thousands) and \( p \) is the unit price (dollars). If the demand is 6000 units, then the price is $8.00 and if the demand is 8000 units, then the price is $6.00.

a. (9) Find the values of \( a \) & \( b \).

b. (4) Find the
demand when the unit price is $9.00.

Solution: a. From the data:

\[ 6 = \sqrt{-64a + b} \implies 36 = -64a + b \]

\[ 8 = \sqrt{-36a + b} \implies 64 = -36a + b \] now subtract the second equation from the first

\[ \implies 28 = 28a \implies a = 1 \]

From the first equation, \[ 36 = -64 + b \implies b = 100 \]

b. \[ p = 9 \implies 9 = \sqrt{-x^2 + 100} \implies 81 = -x^2 + 100 \]

\[ \implies x^2 = 100 - 81 = 19 \implies x = \sqrt{19} = 4.36 \text{ units} \]

7. (20 points) Let \( f(x) = x^2 + x \). a. (13) Compute \( f'(x) \) using the process in our book in chapter 2; b. (7) Find the equation of the tangent line when \( x = -2 \) and put in the \( y = mx + b \) form.

Solution: a. From the definition in the book:

\[ f'(x) = \lim_{h \to 0} \left( \frac{f(-2 + h) - f(-2)}{h} \right) = \lim_{h \to 0} \left( \frac{(-2 + h)^2 + 2 + h - 2}{h} \right) \]

\[ = \lim_{h \to 0} \left( \frac{4 - 4h + h^2}{h} \right) \]

\[ = \lim_{h \to 0} \left( \frac{h^2 - 3h}{h} \right) \]

\[ = \lim_{h \to 0} (h - 3) = -3 \]

b. For the tangent line when \( x = -2 \), slope = \( f'(-2) = -3 \).

A point on the TL is \( x = -2 \) and \( y = f(-2) = 2 \).
Equation of TL is: \( y - 2 = -3 \{ x + 2 \} \); solving for \( y \)

\[ \implies y = -3x - 4 \]