1. (10 points) Solve for $x$; be sure to show your work and do not use your calculator.

(a) $3^{x-x^2} = \frac{1}{3^x}$

Solution: (a) $3^{x-x^2} = 3^{-x} = 3^{-2x}$

$\Rightarrow x - x^2 = -2x \Rightarrow 3x - x^2 = 0$

$\Rightarrow x(3-x)=0 \Rightarrow x = 0 \text{ or } x = 3$

(b) $\ln(x-1) + \ln 4 = \ln(4(x-1))$

$\ln(2x+4) - \ln 2 = \ln \left(\frac{2x+4}{2}\right)$

$\Rightarrow 4(x-1) = x+2 \Rightarrow 4x - 4 = x + 2$

$\Rightarrow 3x = 6 \Rightarrow x = 2$
2. (10 points) The barometric pressure \( p \) (inches of mercury) is related to the altitude \( x \) (in miles) by

\[
p = p(x) = 29.92 e^{-0.02x}
\]

If a balloonist measures the barometric pressure to be 20 inches of mercury, how high up is the balloonist?

Solution: \( p = 20 \), find \( x \)

\[
20 = 29.92 e^{-0.02x} \quad \text{(converted to ln form)}
\]

\[
\frac{20}{29.92} = e^{-0.02x} \quad \Rightarrow \quad \ln \left( \frac{20}{29.92} \right) = -0.02x
\]

\[
\Rightarrow 0.40279 = -0.02x
\]

\[
\Rightarrow x = \frac{0.40279}{-0.02} \approx 20.14 \text{ miles}
\]
#3. (a) The concentration of a drug in a patient's organ at time $t$ (in seconds) is given by

$$C(t) = 0.08 + 0.12 \left( 1 - e^{-0.02t} \right)$$

(b) What is the concentration after $\frac{1}{2}$ minute?

(c) In the long run mean $t \to \infty$

$$\lim_{t \to \infty} e^{-0.02t} = 0$$

Solution (a) $t = 30 \Rightarrow$

$$C(30) = 0.08 + 0.12 \left( 1 - e^{-0.02(30)} \right)$$

$$= 0.08 + 0.12 \left( 1 - 0.549 \right)$$

$$= 0.08 + 0.12 \left( 0.451 \right)$$

$$= 0.08 + 0.054 = 0.134$$