1a. Find the eigenvalues and corresponding eigenvectors to the matrix
\[
A = \begin{bmatrix}
-10 & 4 \\
-24 & 10
\end{bmatrix}
\]

b. Find a \(A^{20}\) (Show all work).

c. Suppose \(A\) is an \(n \times n\). Show that eigenvectors associated with difference eigenvalues are linearly independent.

2a Determine whether or not the set of vectors \(V = \{\vec{x} \in \mathbb{R}^3 : x + y + z = 1\}\) form a vector space (Show work).

b. Given the vectors
\[
\vec{x}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \vec{x}_2 = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}, \quad \vec{x}_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}
\]

find an orthonormal basis for \(S = \text{Span}(\vec{x}_1, \vec{x}_2, \vec{x}_3)\).

c. Find \(S^\perp\) the orthogonal complement of \(S\) in \(\mathbb{R}^3\).

3a. Find a basis for \(\text{Im}A\) and \(\text{Ker}A\) of the following matrix.
\[
\begin{bmatrix}
1 & 1 & 1 & 1 & 0 & -1 \\
2 & 1 & 0 & 3 & 1 & 0 \\
0 & 1 & 1 & 0 & 1 & -2 \\
1 & 1 & 0 & 2 & 1 & 0
\end{bmatrix}
\]

b. Find a basis for the \(\text{Im}A^T\)

c. Suppose that \(\vec{x}_1...\vec{x}_m\) span a subspace \(S\) but are not linearly independent. Describe how you would obtain a basis for \(S\) from these vectors.

4. Consider the system of equations
\[
x - 2y + az = 2 \\
x + y + 3z = b \\
3y + z = 2
\]

a. For which values of \(a\) and \(b\) does this system have a unique solution?

b. For which values of \(a\) and \(b\) does this system have no solutions?

c. For which values of \(a\) and \(b\) does this system have an infinite number of solutions? Sketch the solution set.

5. Show whether the following series converge absolutely, converge conditionally, or diverge.

a. \(\sum_{k=0}^{\infty} (-1)^k \frac{(k!)^2}{(2k)!}\)

b. \(\sum_{k=2}^{\infty} k3^{-k}\)

c. \(\sum_{j=4}^{\infty} (-1)^k \frac{\sqrt{j}}{(j + 1)}\)
6a. Find the power series representation for $\int_0^x \frac{t - \sin t}{t^3} dt$.

b. Find the radius of convergence and interval of convergence of the series

$$\sum_{j=2}^{\infty} \frac{(-1/2)^j}{j^2} (x - 1)^j.$$ 

c. Find the Taylor series for $(1 - 3x)^{-3/4}$ at $x = 0$ and determine its radius of convergence.

7a. Find $P_4$ for $\ln(1 - x)$.

b. $f(x) = \cos 2x$, compute $P_4$ at $x = 0$.

c. For the function in (b) find an upper bound for $R_{n+1}$ for $x \in [0, 3]$.

8a. Find the least squares solution to $A\bar{x} = \bar{b}$ where $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$, and $\bar{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ -1 \end{bmatrix}$

b. Which vector in the $\text{Im}A$ is closest to $\bar{b}$?

9. Determine if the improper integrals are convergent or not. (Show work.)

a. $\int_2^3 \frac{1}{x - 2} \, dx$.

b. $\int_0^1 e^{2x} \, dx$. 