1. Show that

\[ w(z) = i \int_0^z (t + 1)^{-\frac{1}{2}} t^{-\frac{1}{2}} (t - 1)^{-\frac{1}{2}} \, dt, \]

maps the upper half plane into the square with vertices 0, b,ib and b+ib. Determine b. (Show all steps)

2.a. Show that the equation \( z = 2 - e^{-z} \) has exactly one root in the right half plane. Where must this root be?
   
   b. Prove that the equation \( z^3 + 9z + 27 \) has no roots in the disk \( |z| < 2 \)

2.b. Suppose that \( f \) is analytic on \( |z| \leq 1 \) and satisfies \( |f(z)| < 1 \) for \( |z| = 1 \). Prove that the equation \( f(z) = z \) has exactly one solution.

3.a. Show that the mapping \( w = z + 1/z \) maps circles \( |z| = \rho \) (\( \rho \neq 1 \)) onto ellipses.
   
   b. Find the Mobius transformation that maps \((-1, 1, i)\) onto \((1, 2, i)\)

   c. Show that Mobius transformation that take the real line to itself must have real coefficients.