Note Title 1/15/2016

<u>Obs</u>: $f:\{1,\ldots,n\}\to X$ and $g:\{1,\ldots,k\}\to X$ both one-one onto then $k=n$

<u>proof</u>: $g^{-1}\circ f:\{1,\ldots,n\}\longrightarrow\{1,\ldots,k\}$

$$\begin{tikzcd} \{1,\ldots,n\} \arrow[r, "f"] & X \\ \{1,\ldots,k\} \arrow[ur, "g"'] & \end{tikzcd}$$

$g^{-1}\circ f$ is one-one onto then $k=n$

<u>Notation</u> $\#X = \#$ of elements in $X$

<u>Def</u>: if $X=\emptyset$ we say $\#X=0$. We still say that $\emptyset$ is finite

Obs: $X' \subset X$. Then $\# X' \leq \# X$

Def: $X$ is said to be infinite if it is not finite. In this case we write $\# X = \infty$

Obs: If $\# X = \infty$, then $\exists\ f : \mathbb{N} \to X$ that is one-one

proof: $X \neq \emptyset$. Let $x_1 \in X$. Let $f(1) = x_1$.

Let $x_2 \in X - \{x_1\}$. Let $f(2) = x_2$.

Assume we have $f(i) = x_i \in X$ $x_i \neq x_j$ if $i \neq j$ $1 \leq i, j \leq n$

Let $x_{n+1} \in X - \{x_1, \dots, x_n\}$. Let $f(n+1) = x_{n+1}$

Obs: $X$ is infinite $\iff \exists\ X' \subsetneq X$ and $f : X \to X'$

one-one onto

proof: $\Rightarrow$) $f: \mathbb{N} \to X$ one-one. Let $x_n = f(n)$

$Y = X - f(\mathbb{N})$ $\quad X' = Y \cup \{x_2, x_4, x_6, \ldots\}$

$g: X \to X'$ $\quad g(x) = \begin{cases} x & \text{if } x \in Y \\ x_{2n} & \text{if } x = x_n \end{cases}$

$g$ is one-one onto $\quad X' \subsetneq X$

$\Leftarrow$) If $X' \subsetneq X$ and $X$ is finite, then

$\# X' < \# X < \infty$ then $\nexists f : X \to X'$ one-one onto

Chapter II Real numbers

Properties of addition and multiplication

I) $a+b = b+a$ and $a \cdot b = b \cdot a$ commutative

II) $(a+b)+c = a+(b+c)$ associative

$(a \cdot b) \cdot c = a(b \cdot c)$

III) $a \cdot (b+c) = a \cdot b + a \cdot c$ distributive

IV) $\exists$ 0 and 1 $\in \mathbb{R}$, $0 \neq 1$, such that $a+0 = a$ and $a \cdot 1 = a$

V) $\forall a \in \mathbb{R}$ $\exists$ $-a \in \mathbb{R}$ such that $a+(-a) = 0$

$\forall a \in \mathbb{R}\ a \neq 0\ \exists a^{-1} \in \mathbb{R}\quad a \cdot a^{-1} = 1$

Consequences:

F1 $a+b+c+d = ((a+b)+c)+d = (a+b)+(c+d) \ldots\ldots$

same with multiplication

F2 $a \cdot b \cdot c \cdot d = a \cdot c \cdot d \cdot b \ldots\ldots$ same with addition

F3 $a, b \in \mathbb{R}$ given. $\exists!\, x$ such that $\boxed{x+a=b}$

add $-a$ to both sides to get $x+a+(-a) = b+(-a)$

$$x+0 = b+(-a)$$

$$x = b+(-a)$$

Notation $a+(-b) = a-b$