Set Theory

Note Title 1/11/2016

$x \in S$ x belongs to the set S

$x \notin S$ x does not belong to S

X and Y sets. $X = Y$ if ($x \in X$ implies that $x \in Y$) and ($x \in Y$ implies $x \in X$)

Ex $\{1, 2, 5\}$

$\{x : \text{statement}\}$

$\{x : x \text{ is an even number}\}$

Def $X \subset Y$. The set X is included in the set Y. $x \in X \Rightarrow x \in Y$

Example

$\{4, 8, 12, \dots\} = \{\text{positive multiples of } 4\} \subset \{\text{positive even numbers}\}$

Def: $X \not\subset Y$. X not included in Y. There exists $x \in X$ such that $x \notin Y$

Ex: $\{2, 4, 6, 7\} \not\subset \{2, 4, 6, 8, \dots\}$

Def: If $X \subset Y$ we say that X is a subset of Y. X is a proper subset of Y if $X \subset Y$ but $X \neq Y$.

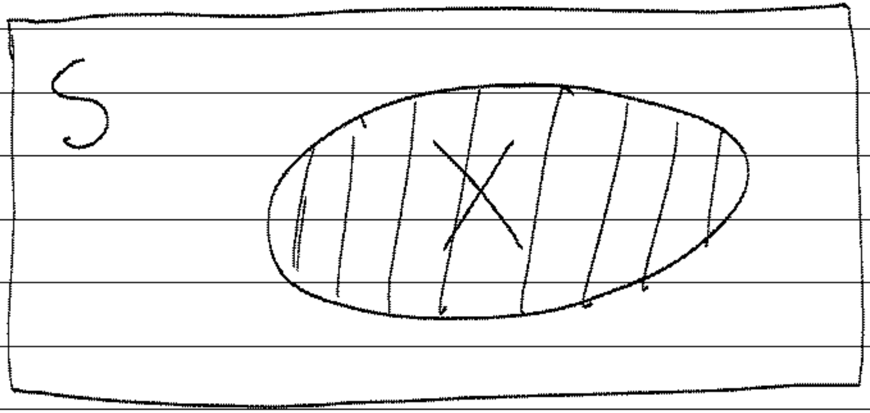
Def: ϕ empty set.

Def: 1) $X \cap Y = \{x : x \in X \text{ and } x \in Y\}$

2) $X \cup Y = \{ x : x \in X \text{ or } x \in Y \}$

3) $X^c = \{ x : x \notin X \}$ complement

$\{ 4, 8, 12, \ldots \}^c$



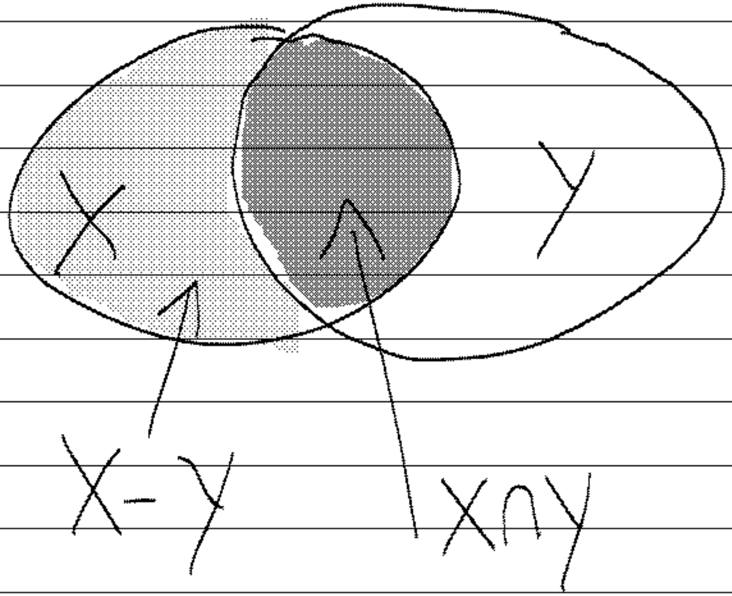
$X^c = \{ x \in S : x \notin X \}$

$S \supset X$

Obs: $(X \cap Y)^c = X^c \cup Y^c$

proof: Let $x \in (X \cap Y)^c \Leftrightarrow x \notin X \cap Y \Leftrightarrow x \notin X$ or $x \notin Y \Leftrightarrow x \in X^c$ or $x \in Y^c \Leftrightarrow x \in X^c \cup Y^c \Rightarrow (X \cap Y)^c = X^c \cup Y^c$

Def: $X - Y = \{x \in X ; x \notin Y\}$



$X \cap Y \cap Z = (X \cap Y) \cap Z$

Let I be a set of indeces.

Ex: $I = \{1,2,3\}$, $I = \{$positive integers$\}$ $I = \mathbb{R} = \{$set of real numbers}

Family of sets $\{X_i ; i \in I\}$ $\{X_i\}_{i \in I}$

<u>Example</u> $X_\lambda = (\lambda, \infty) = \{x \in \mathbb{R} : x \geq \lambda\}$ $\lambda \in \mathbb{R}$

$$\bigcap_{i \in I} X_i = \{x : x \in X_i \text{ for all } i \in I\}$$

$$\bigcup_{i \in I} X_i = \{x : x \in X_i \text{ for an } i \in I\}$$

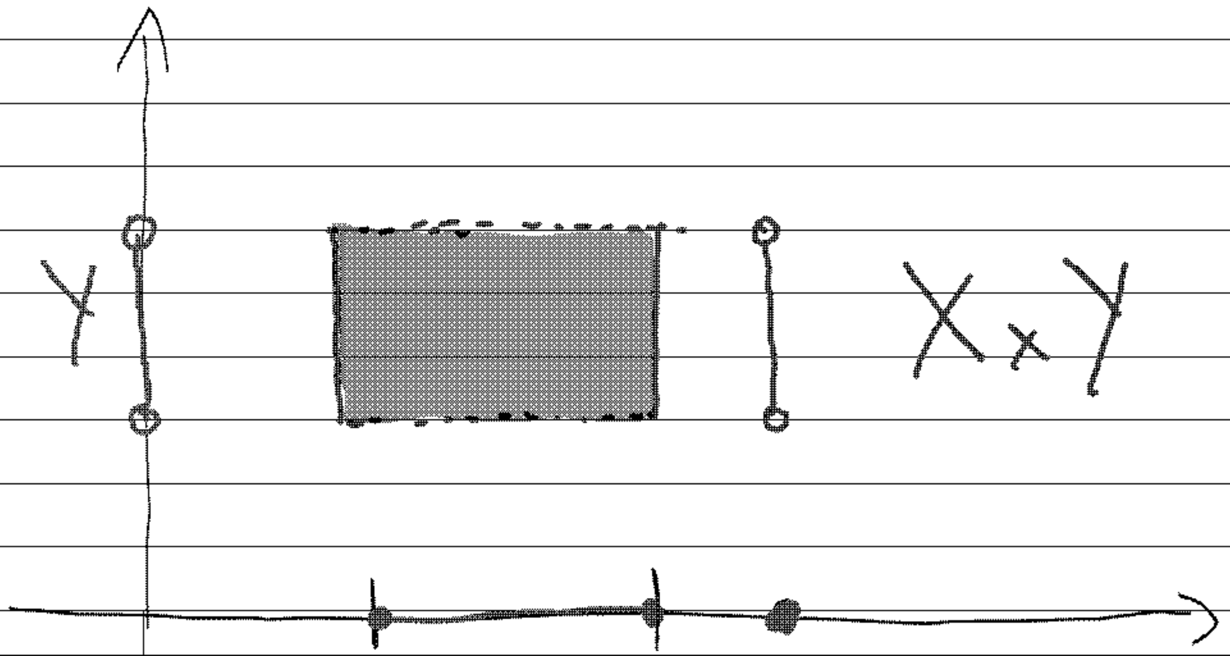
<u>Example</u> $\left(\bigcap_{i \in I} X_i\right)^c = \bigcup_{i \in I} X_i^c$

prove to practice

Cartesian product

$X \times Y = \{(x,y) : x \in X \text{ and } y \in Y\}$

Ex: $\mathbb{R} \times \mathbb{R} = \mathbb{R}^2 = \{(x,y) : x \in \mathbb{R} \text{ and } y \in \mathbb{R}\}$



X

<u>Functions</u> $f: X \longrightarrow Y$

$x \in X \quad f(x) \in Y$

<u>Example</u> 1) $X = \{1,2,3,4,5\}$ $\quad f(1) = 2 \quad f(2) = 3$

$f(3) = 2 \quad f(4) = 5 \quad f(5) = 1$

$Y = \mathbb{R}$

2) $f: \mathbb{R} \to \mathbb{R} \qquad f(x) = x^2 + 2x$

Graph of $f = \{(x, f(x)) : x \in X\} \subset X \times Y$

$f: X \to Y$ $\qquad$ X is called the domain

Y is called the codomain

If X and Y are $\mathbb{R}$