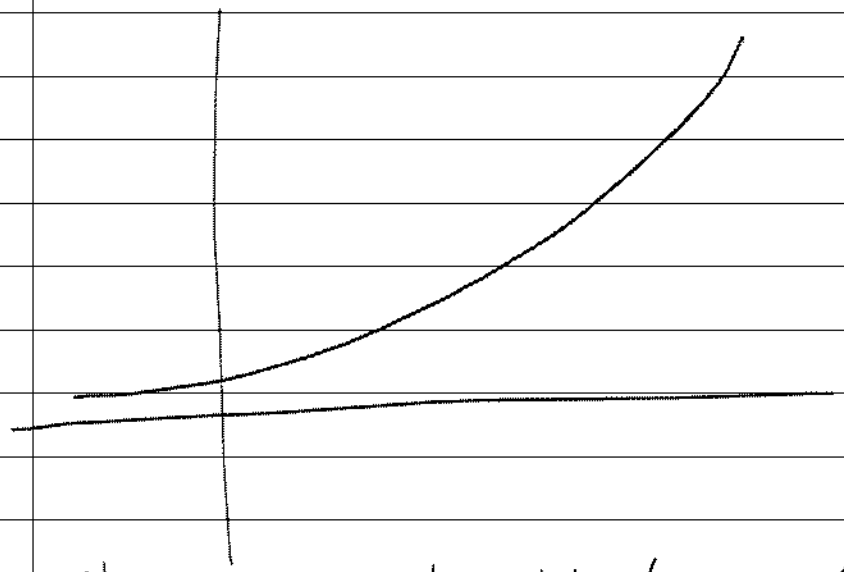
Note Title 4/18/2016

Prop: 1) $\exp(x)$ is continuous



exp is increasing because $\log x$ is increasing and $\exp(x) = \log^{-1} x$

$\exp : \mathbb{R} \to \mathbb{R}_{>0}$

exp is onto, because if $y > 0$, then

$\exp(\log y) = y$.

We proved that, if $f : \mathbb{R} \to \mathbb{R}$ is increasing and $f(\mathbb{R})$ connected

2) $\exp(x)$ is strictly increasing

3) $\exp(\mathbb{R}) = \mathbb{R}_{>0}$

4) $$\lim_{x \to x_0} \frac{\exp(x) - \exp(x_0)}{x - x_0} = \lim_{y \to y_0} \frac{y - y_0}{\log y - \log y_0} =$$

$$\boxed{\begin{array}{l} y = \exp(x) \\ y_0 = \exp(x_0) \end{array}}$$

$$= \frac{1}{\lim\limits_{y \to y_0} \frac{\log y - \log y_0}{y - y_0}} = \frac{1}{\frac{1}{y}} = y = \exp(x_0)$$

5)

$$\log\left(\exp(x+y)\right) = x+y$$

$$\parallel$$

$$\log\left(\exp(x)\exp(y)\right) = \log\exp(x) + \log\exp y = x+y$$

then $\exp(x+y) = \exp(x)\exp(y)$

6) $\exp(x-y) = \dfrac{\exp(x)}{\exp(y)}$

7) $\exp(nx) = (\exp(x))^n \quad \forall\ n \in \mathbb{Z}$

Obs: $\boxed{x^n = \exp(\log(x^n)) = \exp(n \log x)}$ $\forall\, n \in \mathbb{Z}$

Def: $x > 0$ $x \in \mathbb{R}$, $n \in \mathbb{R}$. Then we define $\boxed{x^n = \exp(n \log x)}$

Properties: $x^{n+m} = \exp((n+m)\log x) = \exp(n \log x + m \log x) = \exp(n \log x)\exp(m \log x)$

$$x^{n-m} = \frac{x^n}{x^m}$$

$$(x^n)^m = x^{nm}$$

$$(xy)^n = x^n y^n$$

$$\frac{d\,x^n}{dx} = n\,x^{n-1}$$

<u>Def</u>: $e = \exp(1)$

$$\boxed{e^x} = \exp(x \log e) = \exp(x \log \exp(1)) =$$

$$= \boxed{\exp(x)}$$

Problem 7, Ch5

γ between $f'(a)$ and $f'(b)$. Show $\exists\, c \in (a,b)$ such that $f'(c) = \gamma$. assume $f'(a) < f'(b)$

suppose $\exists x_1, x_2$ s.t $\dfrac{f(x_1) - f(x_2)}{x_1 - x_2} = \gamma \qquad a \le x_1 < x_2 \le b$

By M.V.T., since f is diff'able, $\exists c$

Let $h(x_1, x_2) = \dfrac{f(x_1) - f(x_2)}{x_1 - x_2}$ in $S = \{a \le x_1 < x_2 \le b\}$

S connected. $h : S \to R$ continuous $\Rightarrow h(S)$ connected.

$f'(a) < \gamma < f'(b)$

$\lim_{x \to a} \dfrac{f(a) - f(x_2)}{a - x_2} = f'(a)$ $\quad \exists \quad a < x_2 \le b$ such that

$h(a, x_2)$

$h(S) \ni h(a, x_2) < \gamma$ Similarly $\exists\, a \leq x_1 <$ ~~b~~ such that

$$\gamma < h(x_1, b) \in h(S)$$

Since $h(S)$ is connected, $\gamma \in h(S)$