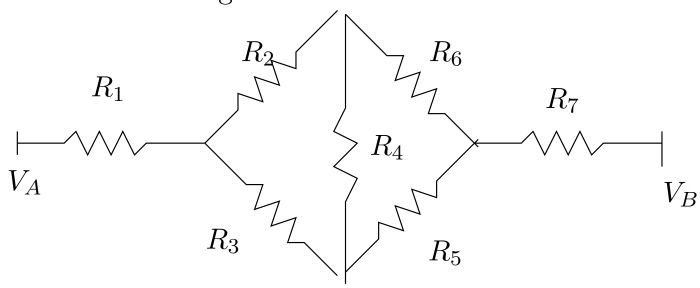
MATH 6643 - Homework 2

Problem Set #1

Do not turn in printouts of your codes. Each problem is worth 5 points.

Problem 1: Consider the following circuit.



The values of the resistances are: $R_1 = 1$, $R_2 = 0.5$, $R_3 = 2$, $R_4 = 0.8$, $R_5 = 3$, $R_6 = 1$ and $R_7 = 1.2$.

Write a program (in the computer) to compute the LU factorization with partial pivoting, a program to solve upper triangular systems and a program to solve lower triangular systems. Use them to compute the effective resistance of the circuit.

Problem 2: If the next operations are done by an ideal computer as described in class, what is the result obtained?

1) The parameters are: base $\beta = 10$, number of digits of precision $t = 3$, and exponent range $L = -20$, $U = 20$

a) $1 + 0.001$

b) $1 - (1 + 0.001)$

c) $(1 - 1) + 0.001$

d) $\Sigma_{i=1}^{20} 10^{-i}$

e) $\Sigma_{i=1}^{2000} 1$

f) $(\Sigma_{i=1}^{1000} 1) + (\Sigma_{i=1001}^{2000} 1)$

Problem 3: Consider the ode

$$\frac{\mathrm{d}^2 y}{\mathrm{d}s^2} = f(s) \quad y(0) = y(1) = 0. \tag{1}$$

A finite difference approximation of (1) leads to

$$x_{i+1} - 2x_i + x_{i-1} = b_i \quad (1 \le i \le n), \tag{2}$$

where $x_0 = x_{n+1} = 0$, $b_i = f(i/(n+1))/(n+1)^2$ and x_i is an approximation of $y(i/(n+1))$.

Let $f(s) = s$.

1) Solve (1) analytically.

2) Write a program (in the computer) to solve (2) with Gaussian elimination with partial pivoting. Compute the solution for $n = 10$, $n = 20$, $n = 40$ and $n = 80$. Plot these solutions and the analytical solution in one figure.

3) How much time the computer took? Could you have predicted the time it took for $n = 80$ once you know the time it took for $n = 20$?

Problem 4: Let $L \in R^{n \times n}$ be lower triangular and nonsingular. Prove that L^{-1} is lower triangular.