

MATH 6643

Problem Set #3.

Problem 1: Let $A \in R^{n \times n}$. Prove that,

$$\|A\|_1 = \max_{1 \leq j \leq n} \sum_{i=1}^n |a_{ij}|. \quad (1)$$

Problem 2: Without calculating their inverse, estimate the condition number of the following matrices. Explain your method.

$$\begin{bmatrix} -6 & -1 & 2 \\ 2 & 5 & 1 \\ 0 & 3 & -4 \end{bmatrix} \quad \begin{bmatrix} 1 & 10 & 2 \\ 2 & 0 & 90 \\ -1 & 3 & 35 \end{bmatrix}. \quad (2)$$

Problem 3: Let $A \in R^{k \times n}$. Prove that for all $U \in R^{k \times k}$ orthogonal and all $V \in R^{n \times n}$ orthogonal. $\|A\|_2 = \|QA\|_2 = \|AV\|_2$

Problem 4: Compute or estimate the 1,2 and ∞ -norm of the following matrices. Explain your method.

$$\begin{bmatrix} -6 & -3 & 2 \\ 2 & 0 & 1 \\ 0 & 3 & -4 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & .2 \\ 2 & 0 & -1 \\ -1 & 3 & 0 \end{bmatrix}. \quad (3)$$