

MATH 6643

Problem Set #4

Do not turn in printouts of your codes.

Problem 1: Write a program that takes as input a vector x and gives as output the Householder reflection $H = I - \beta vv^T$, where $\beta = 2/\|v\|_2^2$ such that $Hx = -\text{sign}(x_1)\|x\|e_1$. Use it for the following values of x :

a) $x^T = [1, 0, -1, 0]$, b) $x^T = [0, 2, 3, 1]$ and c) $x^T = [-1, -1, 2, 0]$.

Report the vectors v and the scalars β only.

Problem 2: Write a program that takes as input a matrix $A \in \mathfrak{R}^{k \times n}$, where $k \geq n$, and gives as output the QR decomposition of A using Householder reflections. Use it for the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ -1 & 0 & 4 \\ 0 & 3 & -1 \end{bmatrix}. \quad (1)$$

Display the Q and R .

Problem 3: What is the polynomial $p(x)$ of degree 3 that minimizes

$$\min_p \sum_{i=1}^m (p(x_i) - f(x_i))^2, \quad (2)$$

where $m = 10$, $x_i = \pi i/m$ and $f(x) = \sin(x)$. (Hint: this is a least square problem). Report how you set up the problem, the algorithm used, and the solution.