

**MATH 6643**

**Practice Quiz #1. The real quiz may contain up to six problems**

**Problem 1:**

a) If  $A \in \mathbb{R}^{n \times n}$  is lower triangular and  $b \in \mathbb{R}^n$ . What is the cost of solving  $Ax = b$  using the algorithm described in class?

b) Does the answer in a) change if  $A$  is upper triangular?

c) Let  $A \in \mathbb{R}^{n \times n}$ . What is the cost of computing the  $LU$  factorization of  $A$  (assuming it can be computed)?

**Problem 2:**

Does the matrix

$$\begin{bmatrix} 0 & -1 & 2 \\ 2 & 5 & 1 \\ 0 & 3 & -4 \end{bmatrix}.$$

have an  $LU$  factorization (without pivoting)? Why?

**Problem 3:**

Are the following statements true or false (justify your answer).

Let  $A, B \in \mathbb{R}^{n \times n}$  and  $C = AB$ , then

a)  $\|C\|_2 = \|A\|_2 \|B\|_2$

b)  $\kappa_2(A) = \kappa_2(B)\kappa_2(C)$

**Problem 4:**

a) Write down the algorithm to compute the  $LU$  factorization (without pivoting).

b) Find the  $LU$  factorization of

$$A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}.$$