

## MATH 6643 - Sample Quiz 2

The actual quiz will have 6 problems

Justify your answers.

### Problem 1:

Let  $A \in \mathbb{R}^{k \times n}$ , where  $k \geq n$ . Suppose that the problem  $\min_x \|Ax - b\|_2$  has at least two solutions. What can you say about the rank of  $A$ ?

### Problem 2:

Let  $A \in \mathbb{R}^{k \times n}$ , where  $k > n$ . Let  $a_j \in \mathbb{R}^k$  be the  $j^{\text{th}}$  column of  $A$ . Suppose  $\{a_1, \dots, a_n\}$  is orthonormal. Suppose also that  $\{v_1, \dots, v_{k-n}\}$  is orthonormal and  $v_i^T a_j = 0$  for all  $i$  and  $j$ . Can you give me a  $QR$  factorization of  $A$ ? (If you can, do it). Can you give me two  $QR$  factorizations of  $A$ ? (If you can, do it).

### Problem 3:

Let  $A \in \mathbb{R}^{3 \times 2}$ . Let  $P$  be the orthogonal projection onto  $\text{ran}(A)$ . Let  $b^T = [-2, 1, 4]$ . Let  $c = Pb$ . Suppose  $c^T = [1, 0, 3]$ . Do we have enough information to know the value of  $y$ , where  $y = \min_x \|Ax - b\|_2$ ? If we do what is it?