Problem 1: Consider the second order ode
\[ x''(s) = 1 \quad 0 < s < 1 \]
\[ x(0) = x(1) = 0. \]

The finite difference discretization of the above equation is
\[ x_{i-1} - 2x_i + x_{i+1} = h^2 \quad x_0 = x_{n+1} = 0 \quad (1 \leq i \leq n) \quad h = 1/(n+1). \]

The criteria stop the algorithm is as in class
\[ \|x^{(k+1)} - x^{(k)}\|_\infty \leq \delta(1 - \rho)\|x^{(k)}\|_\infty. \]

where \( \rho \) is the spectral radius of the corresponding \( B \).

Task 1: Use the Jacobi iteration with \( n = 100 \), \( n = 200 \) and \( n = 400 \) and \( \delta = 0.01 \), \( \delta = 0.01/2 \), \( \delta = 0.01/4 \), \( \delta = 0.01/8 \). Report the number of iterations required and the computing times.

Can you predict the number of iterations and the computing times that we would obtain if \( n = 2000 \) and \( \delta = 0.01/8? \), why? If your answer is yes, predict them.

Task 2: Repeat task 1 but using Gauss-Seidel?

Could you have predicted the number of iterations required from the results obtained with Jacobi? If your answer is yes, predict them.

Task 3: Repeat task 1 but using SOR with the optimal \( \omega \)?

Could you have predicted the number of iterations required from the results obtained with Jacobi? If your answer is yes, predict them.

Task 4: In a same figure plot the exact solution of the ode (solve it analitically) and the numerical solutions with \( n = 100 \), \( n = 200 \), \( n = 400 \) and \( n = 800 \) and \( \delta = 0.01/8 \) obtained with one of the 3 methods.