MATH 6701 - Quiz 2

Justify all your answers. No calculators

**Problem 1 (5 points):** Find the solution of \( 2x = yy'(1 - 2x) \) and \( y(1) = 1 \).

**Problem 2 (6 points):** Let \( y \) be the solution of \( y'(xy - x^2) = y^2 \) and \( y(1) = 2 \). For what value of \( x \) we have \( y(x) = 3? \)

**Problem 3 (6 points):** Find the solution to \( y'' - 2y' + 2y = e^x \cos x, \ y(0) = 1 \) and \( y'(0) = 1 \).

**Problem 4 (6 points):** Find the solution to \( y'' + \frac{2}{x} y' + \frac{2}{x^2} y = 0, \ y(1) = 0 \) and \( y'(1) = 1 \).

**Problem 5 (6 points):** Find the solution to

\[
x' = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix} x
\]

and

\[
x(0) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}
\]

**Problem 6 (6 points):** Find and classify the fixed points of the following system

\[
\begin{align*}
\dot{x} &= x^2 - 2xy \\
\dot{y} &= 2 + x - y
\end{align*}
\]

Make the phase portrait as best as possible.