

Example: Let  $S = \{x \in \mathbb{R}^4 : x_1 + x_2 = 0$   
 $x_3 + 2x_4 = 0\}$ . Find an orthonormal basis

for  $S$ .

solutions:  $S$  is the set of solutions  
of  $x_1 + x_2 = 0$

$$x_3 + 2x_4 = 0$$

We find these solutions using Gaussian  
elimination  $\begin{bmatrix} \boxed{1} & 1 & 0 & 0 \\ 0 & 0 & \boxed{1} & 2 \end{bmatrix}$

$$x = t_1 \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t_2 \begin{bmatrix} 0 \\ 0 \\ -2 \\ 1 \end{bmatrix} \quad t_1, t_2 \in \mathbb{R}$$

this means that

$\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -2 \\ 1 \end{bmatrix} \right\}$  is a basis of  $S$ .

From here, we can find an orthonormal basis using the method you know