Problem 1

$$\left[\begin{array}{cccc|c} 0 & -1 & 1 & 0 & 1 \\ 1 & 2 & 0 & 1 & 2 \\ 1 & -1 & 3 & 1 & 5 \end{array}\right]$$

R_2
R_1

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 2 \\ 0 & -1 & 1 & 0 & 1 \\ 1 & -1 & 3 & 1 & 5 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 2 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & 3 & -3 & 0 & -3 \end{array}\right]$$

$-R_2$
$R_3 - R_1$

$R_1 - 2R_2$
$R_3 - 3R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & 2 & 1 & 4 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$x_1 = 4 - 2t_1 - t_2$$
$$x_2 = -1 + t_1$$
$$x_3 = t_1$$
$$x_4 = t_2$$

$$x = \begin{bmatrix} 4 \\ -1 \\ 0 \\ 0 \end{bmatrix} + t_1 \begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \end{bmatrix} + t_2 \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Problem 2

$$\left[\begin{array}{cccc} 1 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \end{array}\right] \quad \left[\begin{array}{cccc} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & -1 \end{array}\right]$$

$$\left[\begin{array}{cccc} 1 & 0 & -1 & 2 \\ 0 & 1 & 1 & -1 \end{array}\right]$$

$$x = t_1 \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + t_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} \qquad v_2 = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$$u_1 = \frac{v_1}{\|v_1\|} = \begin{bmatrix} 1/\sqrt{3} \\ -1/\sqrt{3} \\ 1/\sqrt{3} \\ 0 \end{bmatrix}$$

$$w_2 = v_2 - (u_1 \cdot v_2)\, u_1$$

$$w_2 = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

$$u_2 = \frac{w_2}{\|w_2\|} = \begin{bmatrix} -1/\sqrt{3} \\ 0 \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$$

Problem 3

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & -1 & 0 \\ 0 & 0 & 2 & -1 & 1 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 1/2 & -1/2 \\ 0 & 1 & 0 & 1/2 & -1/2 & 1/2 \\ 0 & 0 & 1 & -1/2 & 1/2 & 1/2 \end{array}\right] = A^{-1}$$

Problem 4

$$\det \begin{bmatrix} 2-\lambda & -1 \\ 5 & -\lambda \end{bmatrix} = \lambda^2 - 2\lambda + 5 = 0$$

$$\lambda = \frac{2 \pm \sqrt{4-20}}{2} = 1 \pm 2i$$

$\boxed{\lambda = 1 + 2i}$ $\quad A - (1+2i)I = \begin{bmatrix} 1-2i & -1 \\ 5 & -1-2i \end{bmatrix}$

$(1-2i)v_1 - v_2 = 0 \qquad v = \begin{bmatrix} 1 \\ 1-2i \end{bmatrix}$

Eigenvalue	Eigenvector
$1+2i$	$\begin{bmatrix} 1 \\ 1-2i \end{bmatrix}$
$1-2i$	$\begin{bmatrix} 1 \\ 1+2i \end{bmatrix}$

Problem 5 Diagonalize $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$

$$\det \begin{bmatrix} -1-\lambda & 0 & 0 \\ 0 & -\lambda & 2 \\ 0 & 2 & -\lambda \end{bmatrix} = (-1-\lambda)\left[(-\lambda)^2 - 4\right]$$

$\lambda = -1, 2, -2$

$\boxed{\lambda = -1}$ $A + I = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}$ $v = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\boxed{\lambda = 2}$ $A - 2I = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -2 & 2 \\ 0 & 2 & -2 \end{bmatrix}$ $v = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

$\boxed{\lambda = -2}$ $A + 2I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 2 & 2 \end{bmatrix}$ $v = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix} \qquad D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & -1/2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & -1/2 \end{bmatrix}$$

Problem 6 Find the line that best fit the data $\begin{bmatrix}1\\1\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}$

$y = ax + b$

$$\underbrace{\begin{bmatrix}1 & 1\\ 2 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix} = \begin{bmatrix}1\\ 1\\ 2\end{bmatrix}}_{LSP}$$

$$\begin{bmatrix}1 & 2 & 1\\ 1 & 1 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 2 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix} = \begin{bmatrix}1 & 2 & 1\\ 1 & 1 & 1\end{bmatrix}\begin{bmatrix}1\\ 1\\ 2\end{bmatrix}$$

$$\begin{bmatrix}6 & 4\\ 4 & 3\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix} = \begin{bmatrix}5\\ 4\end{bmatrix}$$

$$\left[\begin{array}{cc|c}6 & 4 & 5\\ 4 & 3 & 4\end{array}\right] \qquad \boxed{y = -\tfrac{1}{2}x + 2}$$

$$\left[\begin{array}{cc|c}1 & 2/3 & 5/6\\ 0 & 1/3 & 2/3\end{array}\right] \qquad \begin{bmatrix}a\\ b\end{bmatrix} = \begin{bmatrix}-1/2\\ 2\end{bmatrix}$$

$$\left[\begin{array}{cc|c}1 & 0 & -1/2\\ 0 & 1 & 2\end{array}\right]$$