

TEST III

Time: 50min

Find the general solution of the following differential equations:

1. $y^{(4)} + 5y'' + 4y = 4x^2 + 6$.
2. $y'' + y = \sec x$.
3. $y'' + xf(x)y' + f(x)y = 0$.
4. Let y_1 and y_2 be two solutions of the 2^{nd} order homogeneous equation. Determine whether each of the following statements is true or false and briefly state your reasoning:
 - a. If $w(y_1, y_2) \neq 0$, then y_1 and y_2 cannot have a common zero.
 - b. If y_1 and y_2 have a common max., then $w(y_1, y_2) = 0$.
 - c. If $y_1/y_2 \neq \text{constant}$, then $y_g = c_1y_1 + c_2y_2$.
 - d. If $c_1y_1 + c_2y_2$ is the general solution, then for any given set of initial conditions, c_1 and c_2 are *uniquely* determined.
5. A mass m is attached at the end of a spring in a medium which resists its motion with a force proportional to its speed. What is the maximum viscosity, c , which would allow the spring to vibrate once its equilibrium is disturbed. Assume that the restoring force of the spring is proportional to the displacement and that the stiffness of the spring is k .

Problems 1 through 3 are worth 10pts each, 4 is worth 8pts, and 5 is 12pts.