

TEST IV

Time: 50min

1. Find a particular solution for the following linear differential equations:

$$y^{(4)} - y = 1 - x^3, \quad y'' + 2y' + y = 2x^2e^{-2x} + 3e^{2x}.$$

2. Find the general solution of the following equation (*Hermite's*) and fully justify your answer:

$$y'' - 2xy' + 2py = 0.$$

3. For the following differential equation, verify that the origin is a regular singular point and find two Frobenius series solutions near that point:

$$2xy'' + (x + 1)y' + 3y = 0.$$

Are the series linearly independent? what is their interval of convergence? What is the general solution? Justify your answers.

4. Solve $y' = (1 + x^2)^{-1}$ in two ways and use the results to prove that the following series converges:

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13} - \frac{1}{15} + \cdots$$

What is the limit?

Problems 1 through 4 are worth 16, 12, 12, and 10 points respectively.