TEST IV

1. Find a particular solution for the following linear differential equations:

   \[ y^{(4)} - y = 1 - x^3, \quad y'' + 2y' + y = 2x^2e^{-2x} + 3e^{2x}. \]

2. Find the general solution of the following equation (Hermite's) and fully justify your answer:

   \[ y'' - 2xy' + 2py = 0. \]

3. For the following differential equation, verify that the origin is a regular singular point and find two Frobenius series solutions near that point:

   \[ 2xy'' + (x + 1)y' + 3y = 0. \]

   Are the series linearly independent? What is their interval of convergence? What is the general solution? Justify your answers.

4. Solve \( y' = (1 + x^2)^{-1} \) in two ways and use the results to prove that the following series converges:

   \[
   1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13} - \frac{1}{15} + \cdots
   \]

   What is the limit?

Problems 1 through 4 are worth 16, 12, 12, and 10 points respectively.