Math 100 Oct 7, 1998
Introduction to Mathematical Proof
Fall 1998, UCSC

TEST 1

Time: 120hrs

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1. a) Show that $p \to q$, $\neg q \to \neg p$, and $p \wedge \neg q \to r \wedge \neg r$ are all equivalent. b)Choose a mathematical theorem, or any proposition which you like, and prove it using three different methods: direct, contraposition, and contradiction.

2. Let p and q be a pair of propositions. In how many different ways is it possible to construct a new proposition, say r, whose truth value depends solely on those of p and q. Is it always possible to express r using at most only p, q, negation, disjunction, and parenthesis?

3. a)Let p, q, and r be propositions. Suppose $p \to q$, $q \to r$, and $r \to p$. Show that either p, q, and r are all true or they are all false. b)Prove this phenomenon for the case when we have n propositions.

4. Find three consecutive integers such that the square of the largest one is equal to the sum of the squares of the other two? How many such triples are there? Why?

5. A sequence $\{x_n\}$ is a *Cauchy sequence* provided that for each $\epsilon > 0$, there is a natural number N such that if m, $n > N$, then $|x_n - x_m| < \epsilon$. Without using any negative words, state what it means to say that $\{x_n\}$ is not a Cauchy sequence.

6. Prove or find a counterexample to each of the following:

$$a)(A - B) \cup C = A - (B \cup C), \qquad b)(A' \cup B) \cap (B' \cup C) \subseteq A' \cup C.$$

7. Let A and B be sets. Show that the propositions $A \subset B$, and $A \in B$ are completely independent, i.e., the truth or falsity of one has no bearing on the truth or falsity of the other.

8. Suppose that every nonempty set contains an element which is disjoint from that set. Show that no set can be an element of itself.

9. Prove that for each *even* natural number n,

$$\left(1-\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\cdots\left(1-\frac{(-1)^n}{n}\right)=\frac{1}{2}.$$

10. The Least-Natural-Number Principle states that every nonempty set of natural numbers has a smallest member. Show that this principle implies that every natural number greater than 1 is either a prime number or product of prime numbers.

11. Choose this last problem yourself. Pick something which you like, and find challenging.

Each problem is worth 10 pts.