Math 11B
Calculus with Applications
Fall 1998, UCSC

Jun 7, 1999

FINAL EXAM

Time: 3hrs

1. Evaluate (a) $\int \ln x \, dx$, (b) $\int \frac{e^t}{1+e^t} \, dt$.

2. Find a formula for the volume of a cone of height h and base radius r.

3. One half of a uniform circular disk of radius 1 lies with its diameter along the y-axis and center at the origin. The mass of the half-disk is 1. Find the location of the center of mass $\bar{x}$.

4. Suppose a chain of length $10\,meters$ and total mass of $10\,Kilograms$ is dangling from the top of a building. How much work is required to pull up *half* of the chain. Assume that g=$10\,m/sec^2$.

5. If during an epidemic people get sick at the annual rate of $r(t) = 1000te^{-t/2}$, (a) how many people get sick after two years? (b) How many people get sick altogether?

6. (a) A bank account earns 10% interest compounded continuously. At what (constant, continuous) rate must a parent deposit money into such an account in order to save $100,000 in 10 years for a child's college expenses? (b) If the parent decides instead to deposit a lump sum now in order to attain the goal of $100,000 in 10 years, how much must be deposited now?

7. (a) Sketch the slope field for $y' = x/y$, and draw several solution curves. (b) Solve the differential equation analytically subject to the initial condition that $y = 1$ when $x = 0$.

8. An aquarium pool has volume $100\,liters$. The pool initially contains pure fresh water. At $t = 0$ minutes, water containing $5\,grams/liter$ of salt is poured into the pool at rate of $10\,liters/minute$. The saltwater instantly and totally mixes with fresh water and the excess mixture is drained out of the bottom of the pool at the same rate ($10\,liters/minute$). Let $S(t)$ = the mass of salt in the pool at time t. (a) Write a differential equation for the amount of salt in the pool. (b) Solve the differential equation for $S(t)$. (c) What happens to $S(t)$ as $t \to \infty$?

Turn Over →

9. By looking at the Taylor series, decide which of the following functions is larger for small positive θ. (a) $1 + \sin\theta$, (b) $\frac{1}{1-\theta^2}$. Compute the radius of convergence for each series.

10. Suppose a ball is dropped from an initial height of 10 ft, and each time it bounces, it rises to $\frac{3}{4}$ of the previous height. What is the total vertical distance traveled by the ball?

11 (Bonus). How long does it take for the ball in problem 10 to come to a full stop?

Each problem is worth 10 pts.