Midterm 3

Time: 60min

Note: Justify all your answers.

1. Find $dy/dx$ in one of the following:
   
   \[ a) \quad y = \frac{\sqrt{x + 13}}{(x - 4)(\sqrt{2x + 1})} \quad \text{b) } y = \tan^{-1} x \]

2. Find two of the following integrals
   
   \[ a) \quad \int \frac{1}{x^2 + 2x + 10} \, dx \quad b) \quad \int \cos^5 x \, dx \quad c) \quad \int \tan^{-1} x \, dx \]

3. Find one of the following limits
   
   \[ a) \quad \lim_{x \to 0} (1 + x)^{\frac{1}{x}} \quad b) \quad \lim_{x \to 0} (x^2 \ln x) \]

4. Write the following number as as the ratio of two integers.
   \[ 3.1222222\ldots \]

5. Determine whether or not the following series converge.
   
   \[ a) \quad 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \cdots \quad b) \quad \sum_{n=2}^{\infty} \frac{n}{\ln n} \]
   
   \[ c) \quad \sum_{n=1}^{\infty} \frac{n + 7}{n^2 \sqrt{n}} \quad d) \quad \sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n!} \]

6. Find the convergence set of the power series
   \[ 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots \]
7 (Bonus). Choose one of the following problems:

a) Find the sum of the alternating harmonic series (Hint: Find a power series for \( \ln(x + 1) \) by integrating the power series for \( 1/(1 + x) \)).

a) Find an infinite series which converges to \( \pi \) (Hint: Find a power series for \( 1/(1 + x^2) \) and integrate it to find a power series for \( \tan^{-1} x \)).

c) Find \( \lim_{n \to \infty} \frac{x^n}{n!} \) (Hint: consider the series \( \sum_{n=1}^{\infty} \frac{x^n}{n!} \)).

Problems 2 and 5 are worth 20 points and 40 points respectively; the rest are worth 10 points each