Math 22 Jun 7, 1999
Calculus of Several Variables
Fall 1998, UCSC

FINAL EXAM

Time: 3hrs

1. (a) Compute the curvature of the helix $\langle 2\cos t, 2\sin t, t\rangle$, and (b) find an equation for the tangent line to this curve at $t = 0$.

2. Use differentials to estimate the amount of tin in a closed tin can with diameter 8 cm, height 12 cm, and thickness 0.04 cm.

3. Find maximum and minimum values of $f(x, y) = x^2 + y^2$ subject to the constraint $x^4 + y^4 = 1$.

4. A lamina occupies the part of the disk $x^2 + y^2 \leq 1$ in the first quadrant. Find its center of mass if the density at any point is proportional to its distance from the x-axis.

5. Find the following limits, or show that they do not exist:

$$(a) \lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2 + y^2}, \qquad (b) \lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2 + y^4}.$$

6. Use polar coordinates to evaluate $\int_{-\infty}^{\infty} e^{-x^2}\, dx$.

7. Find the surface area of the portion of the sphere $x^2 + y^2 + z^2 = 1$ which lies above the plane $z = 1/2$.

8. Find the equations of the tangent plane and normal line at the point $(-2, 1, -3)$ to the ellipsoid $x^2/4 + y^2 + z^2/9 = 3$.

9. A rectangular box without lid is to be made of $12\, m^2$ of cardboard. Find the maximum volume of such a box.

10. Find the total mass and the center of mass of the tetrahedron bounded by the coordinate planes, and the plane $x + y + z = 1$, assuming that the density is given by $\rho(x, y, z) = y$.

11 (Bonus). Use spherical coordinates to find the volume of the solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$.

Each problem is worth 10 pts.