

MIDTERM

Time: 70min

1. Find an equation for the plane passing through the points $A(1, 0, 0)$, $B(0, 1, 0)$, and $C(0, 0, 1)$. What is the area of the triangle ABC ?
2. Find the domain and range of each of the following functions and draw their contour graphs showing several level curves:

$$\text{a) } f(x, y) = x^2 - y^2, \quad \text{b) } f(x, y) = \sqrt{1 - x^2 - y^2}.$$

3. Find the position vector of a particle which has the acceleration $\mathbf{a}(t) = \langle 0, 2, 6t \rangle$, initial position $\mathbf{r}(0) = \langle 0, 0, 0 \rangle$, and initial velocity $\mathbf{v}(0) = \langle 1, 0, 0 \rangle$. Draw the shadows (projections) of the path of the particle into the xy , yz , and xz planes.
4. Write an equation in cylindrical and spherical coordinates and identify the surface: $x^2 + y^2 = 2z$. Verify that the point $\langle 1, 1, 1 \rangle$ lies on this surface, and find the coordinates of this point in cylindrical and spherical systems.
5. Compute the curvature of the helix $\langle 2 \cos t, 2 \sin t, t \rangle$, and find an equation for the tangent line to this curve at $t = 0$. Compute the length of one cycle of this curve (e.g., from $t = 0$ to $t = 2\pi$).

Each problem is worth 10 pts.