Math 23B
Multivariable Calculus
Fall 1999, UCSC

Dec 7, 1999

FINAL EXAM

Time: 180min

1. Evaluate $\int\int_D \sin(x^2+y^2)\,dxdy$ where D is the disk $x^2+y^2 \le \pi$.

2. Compute the volume of an ellipsoid with semiaxes a, b, and c.

3. Find the center of mass of the *ice cream cone* given by $x^2+y^2+z^2 \le 1$ and $z \ge \sqrt{x^2+y^2}$, if the density is $\delta(x,y,z) := \sqrt{x^2+y^2+z^2}$.

4. Find the average value of the distance of the *helix* $(\cos t, \sin t, t)$, $0 \le t \le \pi$, from the xz-plane.

5. Find the surface area of the portion of the sphere $x^2+y^2+z^2 = 1$ which lies above the plane $z = \frac{\sqrt{2}}{2}$.

6. Show that the gravitational vectorfield $\mathbf{F} := -\frac{\mathbf{r}}{\|\mathbf{r}\|^3}$ is conservative. What is the total work done by this force in moving a particle from a point $\mathbf{r}_0$ to a point $\mathbf{r}_1$?

7. Compute the area of the region enclosed by the curve $(\cos^3 t, \sin^3 t)$, $0 \le t \le 2\pi$. (*Hint:* use Green's theorem).

8. Show that the volume of any cone with base D and height h is given by $1/3\,Area(D)h$ (*Hint:* suppose that the vertex of the cone is at the origin and apply Gauss's theorem to the vectorfield $\mathbf{r} := (x,y,z)$).

9. Find $\int\int_S \nabla \times \mathbf{F} \cdot d\mathbf{S}$ where S is given by $x^2+y^2+(z-\frac{1}{2})^2 = 1$, and $z \ge 0$, and $\mathbf{F}(x,y,z) := (x+z, y+z, z^2)$ (*Hint:* use Stokes's theorem).

10. Suppose that rain is described by the vectorfield $\mathbf{F}(x,y,z) = -(1,0,1)$. What is the flux through the hemispherical cup $z = -\sqrt{1-x^2-y^2}$? How long does it take before the cup is filled with water?

11 (Bonus). Prove that the there exists a surface which has infinite area but bounds a finite volume.

Each problem is worth 10 points.