Math 23B
Multivariable Calculus
Fall 1999, UCSC

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PRACTICE QUIZ 2

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1. Let $B \subset \mathbf{R}^3$ be a planar region, and let $o \in \mathbf{R}^3$ be a point. If we connect all points in B to o we get a cone, say C, with vertex o and base B. Show that

$$Volume(C) = \frac{1}{3} Area(B) h,$$

where h is the distance of o from the plane of B.

Hints: Follow these steps:

1. Let o be the origin of the coordinte system. Define $\mathbf{r}(x, y, z) := (x, y, z)$. Evaluate the flux of $\mathbf{r}$ through the boundary of C, i.e, $\int\int_{\partial C} \mathbf{r} \cdot \mathbf{n}\, dA$ where $\mathbf{n}$ is the outward unit normal to ∂C.
2. Evaluate the divergence of $\mathbf{r}$ in C, i.e., $\int\int\int_C \nabla \cdot \mathbf{r}\, dV$.
3. Use Gauss's theorem, which states that the total divergence of a vectorfield within a region enclosed by a surface is equal to the flux of that vectorfield through the surface:

$$\int\int\int_C \nabla \cdot \mathbf{r}\, dV = \int\int_{\partial C} \mathbf{r} \cdot \mathbf{n}\, dA.$$