Math 2401 L
Calculus III
Fall 2008, Georgia Tech

Dec 11, 2008

Final

Time: 180 minutes

1. Find the angle between the diagonal of a cube and diagonal of one of its sides.

2. Find the maximum and minimum values of $f(x, y) = x^2 + xy + y^2$ on the disk $x^2 + y^2 \leq 4$.

3. Determine the maximum value of $f(x, y, x) = (xyz)^{1/3}$ given that x, y, z are nonnegative numbers and $x + y + z = k$, k a constant.

4. Find the volume of the "ice cream cone" region bounded inside the sphere $x^2+y^2+z^2=1$ and above the cone $z=\sqrt{x^2+y^2}$.

5. Find the volume of the solid bounded below by the xy-plane and above by the paraboloid $z = 1 - (x^2 + y^2)$.

6. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in two different ways: (i) using change of variables, and (ii) by applying Green's theorem.

7. Find center of mass of a hemisphere of radius 1, i.e., the 2-dimensional surface given by $x^2 + y^2 + z^2 = 1$ and $z \geq 0$ (Note: this is not the same as half a ball).

8. An object moves along the parabola $y = x^2$ from $(0, 0)$ to $(2, 4)$. One of the forces acting on the object is $\mathbf{F}(x, y) := (x + 2y)\mathbf{i} + (2x + y)\mathbf{j}$. Calculate the work done by $\mathbf{F}$.

9. Compute the total flux of the vector field $\mathbf{v}(x, y, z) = (x, 2y^2, 3z^2)$ out of the solid given by $x^2 + y^2 \leq 9$, $0 \leq z \leq 1$.

10. The sphere $x^2 + y^2 + z^2 = a^2$ intersects the plane $x + 2y + z = 0$ in a curve C. Calculate the circulation of $\mathbf{v} = 2y\mathbf{i} - z\mathbf{j} + 2x\mathbf{k}$ about C by using Stokes Theorem.