Math 2401 M
Calculus III
Spring 2012, Georgia Tech

Mar 29, 2012

Midterm 2

Time: 60 minutes

Name:

Each problem is worth 15 *points.*

1. Find a function with gradient $\mathbf{F}(x, y) = 2xy\mathbf{i} + (1 + x^2)\mathbf{j}$.

2. Find a normal vector to the surface $z = x^2 + y^2$ at the point $(1, 1, 2)$, and write an equation for the tangent plane at that point.

3. Find the maximum and minimum values of $f(x, y) = x^2 + xy + y^2$ on the disk $x^2 + y^2 \leq 1$.

4. If an open rectangular box has prescribed surace area S, what dimensions yield the maximum volume?

5. Find the volume of the solid bounded below by the xy-plane and above by the paraboloid $z = 1 - (x^2 + y^2)$.

6. Find the volume of the “ice cream cone” region bounded inside the sphere $x^2+y^2+z^2=1$ and above the cone $z=\sqrt{x^2+y^2}$.

7. Find center of mass of half a ball of radius 1, i.e., the region bounded inside the sphere $x^2 + y^2 + z^2 = 1$ and above the plane $z = 0$.