1. Show that if the diagonals of a parallelogram have the same length then the parallelogram is a rectangle.

2. Find the distance between the point (3, 4, 5) and the plane $2x + y + 3z = 5$.

3. Show that for any collection of $n$ real numbers $x_1, x_2, \ldots, x_n$,
   \[
   \frac{x_1 + x_2 + \cdots + x_n}{n} \leq \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}.
   \]

4. Find the equation of the tangent plane to the surface given by $x^3 + y^3 + z^3 = 7$ at the point $(0, -1, 2)$.

5. Show that if the velocity of a path is always orthogonal to its position vector, that is $\mathbf{x}'(t)$ is orthogonal to $\mathbf{x}(t)$ for all $t$, then the path lies on a sphere centered at the origin.

6. Let $f(x, y) = (x^2 + y^2, x)$ and $g(r, \theta) = (r \cos \theta, r \sin \theta)$. Compute $D(f \circ g)$.

7. Find the length of the helix $\mathbf{x}(t) = (\cos t, \sin t, t)$ for $0 \leq t \leq 2\pi$.

8. (Extra Credit) Show that the orbit of a planet always lies in a plane which passes through the sun.

Each problem is worth 15pts.