Math 2411
Honors Calculus III
Spring 2006, Georgia Tech

Feb 15, 2006

Midterm 1

Time: 50min

1. Show that if the diagonals of a parallelogram have the same length then the parallelogram is a rectangle.

2. Find the distance between the point $(3,4,5)$ and the plane $2x+y+3z=5$.

3. Show that for any collection of n real numbers x_1, $x_2, \dots, x_n$,

$$\frac{x_1+x_2+\cdots+x_n}{n} \leq \sqrt{x_1^2+x_2^2+\cdots+x_n^2}.$$

4. Find the equation of the tangent plane to the surface given by $x^3+y^3+z^3=7$ at the point $(0,-1,2)$.

5. Show that if the velocity of a path is always orthogonal to its position vector, that is $\mathbf{x}'(t)$ is orthogonal to $\mathbf{x}(t)$ for all t, then the path lies on a sphere centered at the orgin.

6. Let $f(x,y)=(x^2+y^2,x)$ and $g(r,\theta)=(r\cos\theta, r\sin\theta)$. Compute $D(f\circ g)$.

7. Find the length of the helix $\mathbf{x}(t)=(\cos t,\sin t,t)$ for $0\leq t\leq 2\pi$.

8. **(Extra Credit)** Show that the orbit of a planet always lies in a plane which passes through the sun.

Each problem is worth 15pts.