Midterm 2

1. Find the center of mass of a half disk of radius 1.

2. Find the volume of the ellipsoid \( \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \) (Hint: Use a change of variables).

3. Find the average distance of a point in a ball of radius 1 from the center of the ball.

4. Find the maximum and minimum of \( f(x, y) = xy \) in the region \( x^2 + y^2 \leq 1 \).

5. Show that for any three real numbers \( x, y, z \),
\[
\sqrt[3]{xyz} \leq \frac{x + y + z}{3}.
\]

(Hint: Maximize \( x^2y^2z^2 \) subject to the constraint \( x^2 + y^2 + z^2 = a^2 \).

Each problem is worth 20pts.

Extra Credit: (5pts) Compute \( \int_{-\infty}^{\infty} e^{-x^2} \, dx \).