Math 426 Apr 14, 2004[1]
Introduction to Modern Geometry
Spring 2004, PSU

Lecture Notes 12

These notes are supposed to be on Gauss-Bonnet theorem, and will be typeset later.

Exercise 1. Show that great circles on a sphere and helices on a cylinder are geodesics.

Exercise 2. Compute the Euler characteristic of a torus.

The following are all simple consequences of the Gauss-Bonnet theorem:

Exercise 3. Show that the sum of the angles in a triangle is π.

Exercise 4. Show that the total geodesic curvature of a simple closed planar curve is 2π.

Exercise 5. Show that the Gaussian curvature of a surface which is homeomorphic to the torus must alwasy be equal to zero at some point.

Exercise 6. Show that the Gaussian curvature of a surface which is homeomorphic to a torus must always be zero at some point.

Exercise 7. Show that a simple closed curve with total geodesic curvature zero on a sphere bisects the area of the sphere.

Exercise 8. Show that there exists at most one closed geodesic on a cylinder with negative curvature.

Exercise 9. Show that the area of a geodesic polygon with k vertices on a sphere of radius 1 is equal to the sum of its angles minus $(k-2)\pi$.

Exercise 10. Let p be a point of a surface M, T be a geodesic triangle which contains p, and α, β, γ be the angles of T. Show that

$$K(p) = \lim_{T\to p} \frac{\alpha+\beta+\gamma-\pi}{Area(T)}.$$

In particular, note that the above proves Gauss's Theorema Egregium.

[1] Last revised: April 15, 2004