Apr 21, 2004[1]

Lecture Notes 13

Here are some review exercises.

Exercise 1. Show that the sum of the angles of a geodesic triangle on a surface of positive curvature is more than π, and on a surface of negative curvature is less than π.

Exercise 2. Show that on a simply connected surface of negative curvature two geodesics emanating from the same point will never meet.

Exercise 3. Show that any compact connected surface without boundary in Euclidean space must have a point of positive curvature.

Exercise 4. Let M be a surface in $\mathbf{R}^3$. Is it always possible to put a local chart $X: U \to M$ such that D_1X and D_2X are orthonormal at every point?

Exercise 5. Show that if a surface in $\mathbf{R}^3$ has a plane of symmetry, then the intersection of that plane with the surface is a geodesic.

Exercise 6. Let M be a surface homeomorphic to a sphere in $\mathbf{R}^3$, and let $\Gamma \subset M$ be a closed geodesic. Show that each of the two regions bounded by Γ have equal areas under the Gauss map.

Exercise 7. Use the Gauss-Bonnet theorem to compute the area of the pseudo-sphere, i.e. the surface of revolution obtained by rotating a tractrix.

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