Choose 5 of the following 8 problems. Write complete answers.

1. Show that if a closed planar curve lies inside a circle of radius \( r \) then its curvature is bigger than or equal to \( 1/r \) at some point.

2. Show that if the curvature of a planar curve is monotone, then it has no self intersections.

3. Compute the curvature and torsion of the helix \((r \cos(t), r \sin(t), pt)\), where \( r \) and \( h \) are constants.

4. Show that if the principal normals of a planar curve all pass through the same point, then the curve is a circle.

5. Show that the tantrix of a closed curve intersects every great circle.

6. Let \( \alpha : I \rightarrow \mathbb{R}^3 \) be a unit speed curve whose torsion never vanishes. Suppose that the binormal vector \( B : I \rightarrow \mathbb{S}^2 \) is known. Show that we can then recover the curvature and torsion of \( \alpha \).

7. Suppose that \( \alpha : I \rightarrow \mathbb{R}^2 \) is a closed curve such that for any constant \( s \), \( \|\alpha(t+s) - \alpha(t)\| \) is constant for all \( t \in I \). Show that \( \alpha \) is a circle.

8. Show that the only closed planar curve of constant curvature is a circle.

Each problem is worth 20pts.