Practice Problems

1. Show that if a closed planar curve lies inside a circle of radius $r$ then its curvature is bigger than or equal to $1/r$ at some point.

2. Show that if the curvature of a planar curve is monotone, then it has no self intersections.

3. Suppose the we have a simple closed curve $\alpha: I \to \mathbb{R}^2$ with curvature $\kappa(t) \leq 1$ for all $t \in I$. Show that $\alpha$ contains a disk of radius 1.

4. Show that if the principal normals of a planar curve all pass through the same point, then the curve is a circle.

5. Show that the tantrix of a closed curve intersects every great circle.

6. Let $\alpha: I \to \mathbb{R}^3$ be a unit speed curve whose torsion never vanishes. Suppose that the binormal vector $B: I \to S^2$ is known. Show that we can then recover the curvature and torsion of $\alpha$.

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8. Show that the curvature and torsion of a curve in Euclidean space are invariant under isometries.

9. Show that the length of any simple closed curve of constant width $w$ is equal to $\pi w$.

10. Suppose that $\alpha: I \to \mathbb{R}^2$ is a closed curve such that for any constant $s$, $\|\alpha(t + s) - \alpha(t)\|$ is constant for all $t \in I$. Show that $\alpha$ is a circle.