Math 550 Dec 12, 2001
Vector Calculus
Fall 2001, USC

FINAL EXAM

Time: 180min

Choose 10 problems:

1. Evaluate $\int\int_D \sin(x^2+y^2)dxdy$ where D is the disk $x^2+y^2 \leq \pi$.

2. Find the center of mass of the icecream cone given by $x^2+y^2+z^2 \leq 1$ and $z \geq \sqrt{x^2+y^2}$ if the density is $\delta(x,y,z) = \sqrt{x^2+y^2+z^2}$.

3. Find the average value of the y coordinate of the semicircle $x^2+y^2=1$, $y>0$.

4. Use greens theorem to compute the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

5. Show that the gravitational vector filed $\mathbf{F} := -\frac{\mathbf{r}}{\|\mathbf{r}\|^3}$. What is the total work done in moving a particle from a point $\mathbf{r_0}$ to a point $\mathbf{r_1}$.

6. Use Gauss's theorem to show that the volume of the cone with base D and height h is given by $1/3Area(D)h$.

7. Show that the area of a surface given by rotating the graph of a function $f(x)$, $a \leq x \leq b$, around x-axis is given by $2\pi\int_a^b f(x)\sqrt{1+f'(x)^2}dx$.

8. Use the previous problem to show that the area of a sphere cut by a pair of parallel planes depends only on the distance between the two planes.

9. Prove that the midpoints of a quadrilateral determine a parallelogram.

10. Suppose that a particle of mass m moves on a path $\mathbf{c}(t)$ in the gravitational vectorfield $\mathbf{F}$ according to Newton's second law: $\mathbf{F}(c(t)) = m\mathbf{c}''(t)$. Show that (a) the angular momentum $h(t) := \mathbf{c}(t)\times\mathbf{c}'(t)$ stays constant in time, and (b) $c(t)\cdot h(t) = 0$. What can we conclude from (a) and (b) with regard to the path of the particle?

11. Find the distance between the lines $\ell_1(t) = t(8,-1,0)+(-1,3,5)$ and $\ell_2(t) = t(0,3,1)+(0,3,4)$.

12. Find $\int\int_S \nabla\times\mathbf{F}\dot{d}\mathbf{S}$ where S is the hemisphere given by $x^2+y^2+z^2=1$ and $\mathbf{F}(x,y,z) := (x+z, y+z, z^2)$ (*Hint*: use Stokes theorem).