Math 550
Vector Analysis
Fall 2001, USC

Oct 11, 2001

Midterm 2

Time: 50min

1. Is $\mathbf{F}(x, y, z) = (yz, xz, xy)$ a gradient vector field?

2. Show that if a particle moves with constant speed, then its velocity and acceleration vectors are orthogonal.

3. Compute the length of $\mathbf{c}(t) := (\cos t, \sin t, t)$ from $t = 0$ to $t = 2\pi$.

4. Show that $\|u \times v\|^2 = \|u\|^2\|v\|^2 - (u \cdot v)^2$.

5. What is the distance between the line $\ell = (2, 3, 1) + t(1, 1, 1)$ and the point $(2, 2, 0)$.

6. Let $\mathbf{r}$ be the vector field given by $\mathbf{r}(x, y, z) = (x, y, z)$ and $r := \|\mathbf{r}\|$. Compute the divergence of the gravitational vectorfield $\mathbf{F} := \frac{\mathbf{r}}{r^3}$.

7. Suppose that a particle of mass m moves on a path $\mathbf{c}(t)$ in the gravitational vectorfield $\mathbf{F}$ according to Newton's second law: $\mathbf{F}(c(t)) = m\mathbf{c}''(t)$. Show that (a) the angular momentum $h(t) := \mathbf{c}(t) \times \mathbf{c}'(t)$ stays constant in time, and (b) $c(t) \cdot h(t) = 0$. What can we conclude from (a) and (b) with regard to the path of the particle?

Each problem is worth 15 points.

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