Nov 9, 2001

Midterm 3

Time: 7days

1. Compute the center of mass of a hemisphere of radius one.

2. Find $\int_{-\infty}^{\infty} e^{-x^2}\,dx$.

3. Compute the volume of the region which lies inside the sphere $x^2 + y^2 + z^2 = 1$ and above the cone $z = \sqrt{x^2 + y^2}$.

4. Let $\mathbf{F} = (z^3 + 2xy)\mathbf{i} + x^2\mathbf{j} + 3xz^2\mathbf{k}$. Find the integral of $\mathbf{F}$ around the unit square with vertices $(\pm 1, \pm 1)$.

5. Find the distance between the lines $\ell_1(t) = t(8, -1, 0) + (-1, 3, 5)$ and $\ell_2(t) = t(0, 3, 1) + (0, 3, 4)$.

6. Let $\mathbf{r}$ be the vector field given by $\mathbf{r}(x, y, z) = (x, y, z)$ and $r := \|\mathbf{r}\|$. Compute the curl of the gravitational vectorfield $\mathbf{F} := \frac{\mathbf{r}}{r^3}$, and show that $\mathbf{F} := -\nabla \frac{1}{r}$.

7. A ring in the shape of the curve $x^2 + y^2 = 1$ has density $\rho(x, y) = |x| + |y|$. What is the mass of the ring.

Each problem is worth 15 points.