Midterm 3

1. Compute the center of mass of a hemisphere of radius one.

2. Find \( \int_{-\infty}^{\infty} e^{-x^2} \, dx. \)

3. Compute the volume of the region which lies inside the sphere \( x^2 + y^2 + z^2 = 1 \) and above the cone \( z = \sqrt{x^2 + y^2} \).

4. Let \( \mathbf{F} = (z^3 + 2xy)i + x^2j + 3xz^2k \). Find the integral of \( \mathbf{F} \) around the unit square with vertices \((\pm 1, \pm 1)\).

5. Find the distance between the lines \( \ell_1(t) = t(8, -1, 0) + (-1, 3, 5) \) and \( \ell_2(t) = t(0, 3, 1) + (0, 3, 4) \).

6. Let \( \mathbf{r} \) be the vector field given by \( \mathbf{r}(x, y, z) = (x, y, z) \) and \( r := ||\mathbf{r}|| \). Compute the curl of the gravitational vectorfield \( \mathbf{F} := \mathbf{r}/r \), and show that \( \mathbf{F} := -\nabla \frac{1}{r} \).

7. A ring in the shape of the curve \( x^2 + y^2 = 1 \) has density \( \rho(x, y) = |x| + |y| \). What is the mass of the ring.

Each problem is worth 15 points.