Midterm 1

1. Find a vector of length 5 that points in the direction opposite to 2\mathbf{i} + \mathbf{j} - \mathbf{k}.

2. Show that the diagonals of a parallelogram have the same length if and only if the parallelogram is a rectangle.

3. Find the area of the triangle with vertices (1, 0, 1), (3, 0, 1), (1, 3, 1).

4. If \mathbf{a} \times \mathbf{b} = \mathbf{i} + \mathbf{j} + \mathbf{k}, what is (2\mathbf{a} + \mathbf{b}) \times (\mathbf{a} - 3\mathbf{b})?

5. Find the distance between the point (2, -1) and the line \ell: x = 3t + 7, y = 5t - 3.

6. What is the distance between the two planes
   \[ 5x - 2y + 2z = 12 \quad \text{and} \quad -10x + 4y - 4z = 8. \]

7. Find a formula for the distance between the planes \(Ax + By + Cz = D_1\) and \(Ax + By + Cz = D_2\).

8. Show that if a path \(x(t)\) lies on a sphere, then \(x(t)\) is always perpendicular to its derivative.

9. Show that for any three real numbers \(a, b, c\), we have:
   \[
   \frac{a + b + c}{3} \leq \sqrt{a^2 + b^2 + c^2}.
   \]

Each problem is worth 12pts.