A NEW VERSION OF THE ISOMORPHIC BUSEMANN-PETTY PROBLEM FOR ARBITRARY FUNCTIONS

We consider the isomorphic Busemann-Petty problem for two different functions, as follows.

**Theorem 1.** Let $K, L$ be star bodies in $\mathbb{R}^n$, let $0 < k < n$, and let $f, g$ be non-negative locally integrable functions on $\mathbb{R}^n$ so that $\|g\|_\infty = g(0) = 1$ and

$$\int_{K \cap H} f \leq \int_{L \cap H} g$$

(1)

for all $(n - k)$-dimensional linear subspaces $H \subset \mathbb{R}^n$. Then

$$\int_K f \leq (d_{ovr}(K, \mathcal{BP}^n_k))^k \frac{n}{n - k} \left| K \right|^\frac{k}{n} \left( \int_L g \right)^\frac{n-k}{n}.$$

Here $d_{ovr}(K, \mathcal{BP}^n_k)$ is the outer volume ratio distance from the body $K$ to the class of generalized $k$-intersection bodies in $\mathbb{R}^n$.

One advantage over previously known results is that the Banach-Mazur distance is replaced by smaller outer volume ratio distance. In particular, this allows to get an absolute constant in the case where $K$ is an unconditional convex body. Also this version immediately implies the slicing inequality for arbitrary functions, similar to the case of volume.

This is joint work with Grigoris Paouris and Artem Zvavitch.